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Rare transitions between metastable states in the stochastic Chaffee–Infante equation.

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We present a numerical and theoretical study of the transitions in the Stochastic one dimensional Chaffee–Infante equation. The one dimensional Chaffee–Infante equation, also know as the Ginzburg–Landau or Allen–Cahn equation in physics, is the prototype equation for bistability in extended systems. As such, it is the perfect model equation for the test of numerical or theoretical methods intended at investigating metastability in more complex stochastic partial differential equations ; typically those arising in oceanicl fluid dynamics. Among other examples, one can think of the alternance of meander paths of the Kuroshio current near Japan, or the switching of the thermohaline circulation in the north Atlantic ocean. The reactive trajectories, the realisations of the dynamics that actually evolve from one metastable state to the other, are the central events in such studies.

The novelty and originality of our approach is the combination of theoretical approaches with a novel numerical method, Adaptive Multilevel Splitting (AMS), for the computation of the full distribution of reactive trajectories and all the properties of the rare transitions. AMS is a mutation selection/selection algorithm that uses N clones dynamics of the system of interest, and only requires $N|\ln(\alpha)|$ iterations. Meanwhile several $1/\alpha$ realisations are required for a direct numerical simulation (with α the probability of observing a transition). It thus becomes a very powerful method when the noise amplitude and therefore α goes to zero.

We used the algorithm to compute the properties (escape probability, mean first passage time, average duration of reactive trajectories, number of fronts *etc.*) of the transition in the full parameter space (L, β) (with L the size of the system and β the inverse of the noise amplitude).

There is an excelent quantitative agreement with the various theoretical approaches of the study of metastability. All of them are asymptotic and therefore concern only specific sections of the (L, β) plane. In the low noise limit, the reactive trajectories can be deduced from the Freidlin–Wentzel principle of large deviations (that yields the instanton trajectories), while the mean first passage times are derived using the Eyring–Kramers theory. However, these approaches require that the saddles between the metastable states are not to flat, *i.e.* that the size of the system is not too large. In the large L limit, we are still able to calculate the mean first passage times using an explicit description of the system. In the large noise limit, where the instantons are no longer the most probable trajectories, we can predict the number of fronts of the trajectories.

The combined numerical and theoretical approaches have proven their efficiency in this system and are therefore very promising tools for the study of more complex systems. Indeed, the simplest space-dependent models of the thermohaline circulation do have a relatively simple phase space. However, this is not necessarily the case of the shallow-water or quasi-geostrophic models which are the simplest way of describing currents like the Kuroshio.