



## **Fundamental solution of Laplace's equation in oblate spheroidal coordinates and Galerkin's matrix for Neumann's problem in Earth's gravity field studies**

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In this paper the reciprocal distance is used for generating Galerkin's approximations in the weak solution of Neumann's problem that has an important role in Earth's gravity field studies. The reciprocal distance has a natural tie to the fundamental solution of Laplace's partial differential equation and in the paper it is represented by means of an expansion into a series of oblate spheroidal harmonics. Subsequently, the gradient vector of the reciprocal distance is constructed. In the computation of its components the expansion mentioned above is employed. The paper then focuses on the scalar product of reciprocal distance gradients in two different points and in particular on a series representation of a volume integral of the scalar product spread over an unbounded domain given by the exterior of an oblate spheroid (oblate ellipsoid of revolution). The integral yields the entries of Galerkin's matrix. The numerical interpretation of all the expansions used as well as the respective software implementation within the OpenCL framework is treated, which concerns also a numerical evaluation of Legendre functions of a real and an imaginary argument. In parallel an approximate closed formula expressing the entries of Galerkin's matrix (with an accuracy up to terms multiplied by the square of numerical eccentricity) is derived for convenience and comparison. The paper is added extensive numerical examples that illustrate the approach applied and demonstrate the accuracy of the derived formulas. Aspects related to practical applications are discussed.