Spatiotemporal waves caused by gravitational instability of an arbitrary stratified elasto-viscous rock massif

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The cause of the spatially-temporal wave-like out-of-plane distortions of originally horizontal layers of the earth’s crust is still one of the most important and debated problems in solid earth physics. Are these distortions originated due to action of the high horizontal compressions applied to a massif or, contrary, they are the result of spontaneous deformation of the massif under gravity without action of another pronounced external agents? The second possibility is called the gravity instability. Understanding of the geodynamic evolution of the region is crucially dependent on what the answer will be chosen from the above alternative. Lord Rayleigh (1883) was the first who solved the problem of gravity instability for the system of 2 infinite layers of ideal incompressible fluids and showed that the system is gravitationally unstable when the upper fluid is denser than the lower one (i.e. the density inversion exists). Since then numerous attempts have been made to generalise the properties of constituents. 1961 till now were the years of unsuccessful attempts to take into account the elastic properties of fluids.

We used the principally new approach to the stability analysis of a system which is a bounded 3D multi-layered fluid domain possessing cross-sections of arbitrary shape and prescribed boundary conditions at all parts of the boundary. It is assumed that the fluid is arbitrary stratified according to both density $\rho$ and nonlinear elastic properties. By using a static energy criterion for stability/instability alongside with the reference (“Lagrangian”) description of a continuum we succeeded in solving the problem completely, namely, we obtained the necessary and sufficient condition for stability (violation of which is the necessary and sufficient condition for instability). It was found that the system under study is unstable if and only if 1) density inversion exists at least at one interface between the fluids and/or 2) there exists a depth range $h+dh$ for which the actual increase of $\rho$ is less than corresponding increase for hypothetical homogeneous fluid provided that both possess equal densities anywhere inside $h+dh$. Both assertions are not obvious and are proved by constructing the corresponding instability modes (IMs) satisfying the kinematic boundary conditions. If the latter are of adhesion type, the constructed IMs are suitable for a viscous fluid too. That allows to extend the results to the case of elastic-viscous fluids, for which the very that type of kinematic boundary conditions takes place. In the case of instability (both for an ideal elastic and for an elastic-viscous fluid) we obtained lower bounds for the greatest exponent of the IMs growth. Not less important for applications (in particular the geophysical ones) is the case when one or both layers are formed not by fluids, but by solid elastic materials. Thus, obtained above the necessary and sufficient conditions of stability for the system of fluids in the case of solid elastic materials appear to be sufficient but not necessary; it means that their violation is not a sufficient condition for instability.