Geophysical Research Abstracts
Vol. 17, EGU2015-2728-1, 2015
EGU General Assembly 2015
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On the Stability of Wave Disturbances in Non-Pressure Round-Cylindrical Channels

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In hydraulic engineering practice, is well know of and take into account the fact that for a nearly fully filled gravity-flow tunnel with a circular cross section the water flows with shocks, i.e. unstable. Such a phenomenon also occurs when emptying a bottle, but no mathematical confirmation has so far been found for it. In the paper, the estimate of the flow stability is obtained for two limiting cases: - when the channel of circular cross-section is nearly fully filled with water and when it is nearly empty, i.e. the water flow in the channel has a small depth as compared with the radius of the water conduit.

Wave equations written in a cylindrical system of coordinates x, r, θ where the x- axis coincides with the axis of the channel; r is the radius vector, θ is the angle counted off from the equatorial plane of the channel upward (with sign „+") and downward (with sign „-") are simplified by neglecting the change of the polar angle $(\frac{\pi}{2} - \theta)$in limit of a small width of the free surface of the flow. As a result of this simplification the Helmholtz equation for the wave potential reduces to a Bessel equation with respect to the function $\psi(r)$ not depending on the angle θ and its asymptotic solution will be expressed by the relation

$$\psi(r) = C\sqrt{\frac{R_0}{r}}\cosh k(R_0 - r). \tag{1}$$

Dispersion relations will take the form

$$\sigma = kU_0 \pm i\sqrt{gk\tanh k(R_0 - h)} \tag{2}$$

- for channels with nearly full filling, and

$$\sigma = kU_0 \pm \sqrt{gk\tanh k(R_0 - h)} \tag{3}$$

- for round-cylindrical channels with a small water depth**.**

In these relations, R_0 is the radius of the channel, U_0 is the stationary water flow velocity, i is the imaginary unit, h is the distance between the horizontal axis and the water level in the channel, σ is the wave disturbance frequency, k is the wave number, C is an arbitrary constant.

In the first case, the relation (2) indicates the occurrence of Helmholtz instability of wave disturbances independently of a velocity value of stationary water flow. This result fully agrees with the results of thorough experimental studies of Chanishvili (1947), according to which the occurrence of flow instability in gravity-flow conduits of large diameter does not depend either on the flow velocity or on the presence of an air layer between the water free surface and the channel vault – this instability always appears when the channel is filled up to 92-93% of its height (diameter).

In the other limiting case, according to (3), the wave motion in a channel of circular cross-section with a small water depth is always stable.

Reference

1. Chanishvili, A.G. A Steady Stream of Water in the Free-flow Pipelines. Proceedings TNISGEI, Tbilisi, 1947, no 1, pp. 69-85, (in Russian)