



Rogue waves and NLSE Lie point symmetries

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In past decades rogue waves have been reported to be the main cause of shipping incidents. The unexpectedness and sudden appearance can be seen as their trait more characteristic. Rogue wave damages are linked with this unexpectedness. Therefore many studies have been carried out to ascertain the possible mechanisms of generation of rogue waves. Since the pioneering work of Zakharov researchers have found the so called Nonlinear Schrödinger Equation as the source of solutions to different kinds of rogue waves, Akhmediev, Peregrine, Matveev and many others.

Following the well-known Lie group theory many researchers found the Lie point symmetries of the NLSE. Invariants of this equation are the scaling transformations, Galilean transformations and phase transformations. There are different approaches, which mathematical treatment is outside the scope of this work, but at the end, in a travelling frame, after preserving the Hamiltonian structure we get an ordinary differential equation that is the Duffing equation (well-known as a model of nonlinear oscillations). The next step is the qualitative analysis of this equation. Solutions of the Duffing equation for different coefficients can be put as Jacobi elliptic functions. In particular, in the case of the focusing NLSE, we are concerned with the instabilities, in the sense of Lyapunov, of the transition between some of the solutions. We thought that these instabilities could be the origin of some kind of rogue waves.