



Generalized functional formulation for multi-fractal representation of basin hydraulic geometry

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Natural rivers exhibit power-functional variability in their width, depth, and velocity with flow discharge (*Leopold and Maddock, 1953*). This relation named hydraulic geometry has been empirically supported by many field studies across the world (e.g., *Leopold et al., 1964; Stall and Fok, 1968*). The relationship appears either at a fixed cross-section, showing temporal variability, or along a downstream direction across an entire basin, showing spatial variability, the latter named downstream or basin hydraulic geometry. Theoretical studies that attempt to explain the power-law phenomenon (fractal), have assumed that the watershed is homogeneous hydrologically and geologically.

Nevertheless, real watersheds are often subject to spatially heterogeneous conditions, due to various reasons including partial area storm coverage (*Sólyom and Tucker, 2004*) and transmission losses on bed and banks (*Lane et al., 1997*). In this setting, hydraulic geometry relationships are likely to deviate from monotonic power-law relationship and to follow rather more complex multi-fractal characteristics. In fact, deviation from single power-law was reported for at-a-station relationship of midwest rivers in US (*Dodov and Fofoula-Georgiou, 2004*). In the case of downstream variation, we identify significant multi-fractal characteristics over the Colorado River basin where strong heterogeneity in geological and hydrological settings presents. Conventional power-law hydraulic geometry relationships cannot express the functional variability for these cases.

Motivated by this fact, we generalize the hydraulic geometry functional formulation in this study to express multi-fractal relationships. To do so, we couple the formulation of *Paik and Kumar (2004)*, which generalized at-a-station and downstream relationships, with the formulation of *Dodov and Fofoula-Georgiou (2004)* which was proposed for multi-scaling in at-a-station relationship. The proposed formulation is successfully evaluated with the case of Colorado River basin. This study has potential to broaden our perspective on hydraulic geometry.

Keywords: Hydraulic geometry; Multi-fractal; Heterogeneity; Colorado River

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