



Caustics of atmospheric waves

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Much like light and sound, acoustic-gravity waves in inhomogeneous atmosphere often have a caustic or caustics, where the ray theory predicts unphysical, divergent values of the wave amplitude and needs to be modified. Increase of the wave magnitude in the vicinity of a caustic makes such vicinities of primary interest in a number of problems, where a signal needs to be separated from a background noise. The value of wave focusing near caustics should be carefully quantified in order to evaluate possible nonlinearities promoted by the focusing. Physical understanding of the wave field in the vicinity of a caustic is also important for understanding of the wave reflection from and transmission (tunneling) through the caustic. To our knowledge, in contrast to caustics of acoustic, electromagnetic, and seismic waves as well as gravity waves in incompressible fluids, asymptotics of acoustic-gravity waves in the vicinity of a caustic have never been studied systematically.

In this paper, we fill this gap. Atmospheric waves are considered as linear acoustic-gravity waves in a neutral, horizontally stratified, moving ideal gas of variable composition. Air temperature and wind velocity are assumed to be gradually varying functions of height, and slowness of these variations determines the large parameter of the problem. The scale height of the atmosphere can be large or small compared to the vertical wavelength. It is found that the uniform asymptotics of the wave field in the presence of a simple caustic can be expressed in terms of the Airy function and its derivative. As for the acoustic waves, the argument of the Airy function is expressed in terms of the eikonal calculated in the ray, or WKB, approximation. The geometrical, or Berry, phase, which arises in the consistent WKB approximation for acoustic-gravity waves, plays an important role in the caustic asymptotics. In the uniform asymptotics, the terms with the Airy function and its derivative are weighted by cosine and sine of the Berry phase, respectively. Explicit expressions are found for the amplitude factors in the dominant term of the uniform asymptotics. The amplitude factors can be expressed in terms of the Berry phase and the divergent wave amplitudes, which are found in the first WKB approximation, but remain finite at the caustic and in its vicinity. The uniform asymptotic expansion of acoustic-gravity waves in the presence of a caustic reduces to known results in the acoustic limit. Physical meaning and corollaries of the newly derived caustic asymptotics will be discussed.