



Mathematics of tsunami: modelling and identification

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Tsunami (long waves in the deep water) motion caused by underwater earthquakes is described by shallow water equations

$$\begin{cases} \eta_{tt} = \operatorname{div}(gH(x, y) \cdot \operatorname{grad} \eta), & (x, y) \in \Omega, t \in (0, T); \\ \eta|_{t=0} = q(x, y), \quad \eta_t|_{t=0} = 0, & (x, y) \in \Omega. \end{cases} \quad (1)$$

Bottom relief $H(x, y)$ characteristics and the initial perturbation data (a tsunami source $q(x, y)$) are required for the *direct simulation of tsunamis*. The main difficulty problem of tsunami modelling is a very big size of the computational domain ($\Omega = 500 \times 1000$ kilometres in space and about one hour computational time T for one meter of initial perturbation amplitude $\max |q|$). The calculation of the function $\eta(x, y, t)$ of three variables in $\Omega \times (0, T)$ requires large computing resources. We construct a new algorithm to solve numerically the problem of determining the moving tsunami wave height $S(x, y)$ which is based on kinematic-type approach and analytical representation of fundamental solution. Proposed algorithm of determining the function of two variables $S(x, y)$ reduces the number of operations in 1.5 times than solving problem (1). If all functions does not depend on the variable y (one dimensional case), then the moving tsunami wave height satisfies of the well-known Airy-Green formula: $S(x) = S(0) \sqrt[4]{H(0)/H(x)}$.

The problem of identification parameters of a tsunami source using additional measurements of a passing wave is called *inverse tsunami problem*. We investigate two different inverse problems of determining a tsunami source $q(x, y)$ using two different additional data: Deep-ocean Assessment and Reporting of Tsunamis (DART) measurements and satellite altimeters wave-form images. These problems are severely ill-posed. The main idea consists of combination of two measured data to reconstruct the source parameters. We apply regularization techniques to control the degree of ill-posedness such as Fourier expansion, truncated singular value decomposition, numerical regularization. The algorithm of selecting the truncated number of singular values of an inverse problem operator which is agreed with the error level in measured data is described and analysed.

In numerical experiment we used conjugate gradient method for solving inverse tsunami problems. Gradient methods are based on minimizing the corresponding misfit function. To calculate the gradient of the misfit function, the adjoint problem is solved. The conservative finite-difference schemes for solving the direct and adjoint problems in the approximation of shallow water are constructed. Results of numerical experiments of the tsunami source reconstruction are presented and discussed. We show that using a combination of two types of data allows one to increase the stability and efficiency of tsunami source reconstruction.

Non-profit organization WAPMERR (World Agency of Planetary Monitoring and Earthquake Risk Reduction) in collaboration with Institute of Computational Mathematics and Mathematical Geophysics of SB RAS developed the Integrated Tsunami Research and Information System (ITRIS) to simulate tsunami waves and earthquakes, river course changes, coastal zone floods, and risk estimates for coastal constructions at wave run-ups and earthquakes. The special scientific plug-in components are embedded in a specially developed GIS-type graphic shell for easy data retrieval, visualization and processing. We demonstrate the tsunami simulation plug-in for historical tsunami events (2004 Indian Ocean tsunami, Simushir tsunami 2006 and others).

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