



## A research on solutions of a wave equation of shallow water for roll waves of debris flow

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Intermittent surges of debris flow are observed in mountains region in Europe, Aisia and others. A purpose of this research is to obtain characteristic of wave equation on shallow water for debris flow surges.

Considering a flow in a rectangular straight channel, where the width is very large compared to a flow depth, momentum correction factor  $\beta = 1$ , constant friction factor over mean depth  $h_0$ , a channel slope  $\tan \theta < 1$ , Froude number  $F_r > 1$ , and a long wave condition by results of observations and experiments, a wave equation is obtained with

$$\frac{\partial \eta'}{\partial \tau'} + a_1 \eta' \frac{\partial \eta'}{\partial \xi'} - a_2 \frac{\partial^2 \eta'}{\partial \xi'^2} + a_3 \frac{\partial^3 \eta'}{\partial \xi'^3} = 0 \quad (1)$$

where,  $a_1 = (3/2)c_0'^2$ ,  $a_2 = (1/2) \left( 1/c_0'^2 - 1/2 \right) \tan \theta (c_0'/u_0')$ ,

$a_3 = (1/2) \left\{ (2 + c_0'^4)/(2c_0'^2) - 3/2 \right\}$ ,

and  $\eta'$ : fluctuation of mean flow depth,  $h_0$ : mean depth,  $h = h_0 + \eta'$ : flow depth,  $\eta' = \eta/h_0$ ,  $x$ : coordinate axis of flow direction,  $x' = x/h_0$ ,  $\xi = \epsilon^{1/2}(x - v_{p0})$ ,  $\xi' = \xi/h_0$ ,  $v_{p0}$ : phase velocity, the velocity parameter of Gardner - Morikawa transformation,  $y$ : coordinate axis of depth direction,  $y' = y/h_0$ ,  $t$ : time,  $t' = t v_{p0}/h_0$ ,  $\tau = \epsilon^{3/2}t$ ,  $\tau' = (v_{p0}/h_0)\tau$ ,  $g$ : acceleration due to gravity,  $\theta$ : slope angle of the channel,  $c_0 = \sqrt{gh_0 \cos \theta}$ : wave velocity of a long wave,  $c_0' = c_0/v_{p0}$ ,  $u_0$ : mean velocity,  $u_0' = u_0/c_0$ .

Using for  $v_{p0} = c_0$  under a long wave condition by observations and experiments, above equation is expressed as

$$\frac{\partial \eta'}{\partial \tau'} + \frac{3}{2} \eta' \frac{\partial \eta'}{\partial \xi'} - \frac{1}{4} \frac{\tan \theta}{u_0'} \frac{\partial^2 \eta'}{\partial \xi'^2} = 0. \quad (2)$$

This equation is a kind of Burgers equation. Analytical solutions for different wave number  $k = 1/2, 3/2, 5/2$  and  $k = 1, 2, 3$  on initial conditions were obtained, and calculated by numerical analysis. These results show that the wave shape are deformed to a wave of wave number  $k = 1$  for not multiple wave number. This indicates that a surge is formed with a wave length from the wave of a lot of wave numbers in initial state on actual surges or experimental surge flows.