



## **Topological inversion for solution of geodesy-constrained geophysical problems**

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Geodetic data, mostly GPS observations, permit to measure displacements of selected points around activated faults and volcanoes, and on the basis of geophysical models, to model the underlying physical processes. This requires inversion of redundant systems of highly non-linear equations with  $>3$  unknowns; a situation analogous to the adjustment of geodetic networks.

However, in geophysical problems inversion cannot be based on conventional least-squares techniques, and is based on numerical inversion techniques (a priori fixing of some variables, optimization in steps with values of two variables each time to be regarded fixed, random search in the vicinity of approximate solutions). Still these techniques lead to solutions trapped in local minima, to correlated estimates and to solutions with poor error control (usually sampling-based approaches).

To overcome these problems, a numerical-topological, grid-search based technique in the  $R^N$  space is proposed ( $N$  the number of unknown variables). This technique is in fact a generalization and refinement of techniques used in lighthouse positioning and in some cases of low-accuracy 2-D positioning using Wi-Fi etc. The basic concept is to assume discrete possible ranges of each variable, and from these ranges to define a grid  $G$  in the  $R^N$  space, with some of the gridpoints to approximate the true solutions of the system. Each point of hyper-grid  $G$  is then tested whether it satisfies the observations, given their uncertainty level, and successful grid points define a sub-space of  $G$  containing the true solutions. The optimal (minimal) space containing one or more solutions is obtained using a trial-and-error approach, and a single optimization factor. From this essentially deterministic identification of the set of gridpoints satisfying the system of equations, at a following step, a stochastic optimal solution is computed corresponding to the center of gravity of this set of gridpoints. This solution corresponds to a minimum variance numerical solution and requires only forward computations.

The efficiency of the topological inversion technique was tested in the adjustment of conventional geodetic networks and in truth-based approaches (inversion of systems of equations with synthetic data produced by known values of variables) for up to about 20 variables and grids with  $10^{10}$  gridpoints.