



High-order Hybridized Discontinuous Galerkin (HDG) method for wave propagation simulation in complex geophysical media (elastic, acoustic and hydro-acoustic); an unifying framework to couple continuous Spectral Element and Discontinuous Galerkin Methods

Sebastien Terrana (1,2), Jean-Pierre Vilotte (1), Laurent Guillot (2), and Christian Mariotti (2)

(1) Institut de Physique du Globe de Paris, Équipe de Sismologie, (2) Commissariat à l'Énergie Atomique, DAM, DIF, F-91297 Arpajon, France

Today seismological observation systems combine broadband seismic receivers, hydrophones and micro-barometers antenna that provide complementary observations of source-radiated waves in heterogeneous and complex geophysical media. Exploiting these observations requires accurate and multi-physics – elastic, hydro-acoustic, infrasonic - wave simulation methods.

A popular approach is the Spectral Element Method (SEM) (Chaljub et al, 2006) which is high-order accurate (low dispersion error), very flexible to parallelization and computationally attractive due to efficient sum factorization technique and diagonal mass matrix. However SEMs suffer from lack of flexibility in handling complex geometry and multi-physics wave propagation. High-order Discontinuous Galerkin Methods (DGMs), i.e. Dumbser et al (2006), Etienne et al. (2010), Wilcox et al (2010), are recent alternatives that can handle complex geometry, space-and-time adaptativity, and allow efficient multi-physics wave coupling at interfaces. However, DGMs are more memory demanding and less computationally attractive than SEMs, especially when explicit time stepping is used.

We propose a new class of higher-order Hybridized Discontinuous Galerkin Spectral Elements (HDGSEM) methods for spatial discretization of wave equations, following the unifying framework for hybridization of Cockburn et al (2009) and Nguyen et al (2011), which allows for a single implementation of conforming and non-conforming SEMs. When used with energy conserving explicit time integration schemes, HDGSEM is flexible to handle complex geometry, computationally attractive and has significantly less degrees of freedom than classical DGMs, i.e. the only coupled unknowns are the single-valued numerical traces of the velocity field on the element's faces. The formulation can be extended to model fractional energy loss at interfaces between elastic, acoustic and hydro-acoustic media.

Accuracy and performance of the HDGSEM are illustrated and assessed against classical SEMs and DGMs through two-dimensional geophysical examples, as well as the coupling between hybridized SEMs and DGMs in the same computational domain. Finally on-going extension of this HDG method for three-dimensional wave propagation phenomena will be outlined in conclusion.

References:

- Chaljub, E., Komatitsch, D., Vilotte, J.-P., Capdeville, Y., Valette, B., Festa, G., Spectral Element analysis in seismology, *Advances in Geophysics*, 48, 365-419, 2006.
- Cockburn, B., Gopalakrishnan, J., Lazarov, R., Unified hybridization of discontinuous Galerkin, mixed and continuous Galerkin methods for second order elliptic problems, *SIAM J. Num. Anal.*, 47, 2, 1319-1365, 2009.
- Dumbser, M. and M. Käser, An arbitrary high-order discontinuous Galerkin method for elastic waves on unstructured meshes II : the three-dimensional isotropic case, *Geophys. J. Int.*, 167 (1), 319-336, 2006.
- Etienne, V., Chaljub, E., Virieux, J., Glinsky N., A hp-adaptative Discontinuous Galerkin finite element method for 3-D elastic wave modelling, *Geophys. J. Int.*, 183, 941-962, 2010.
- Komatitsch, D. and J.-P. Vilotte, The Spectral Element Method : An efficient tool to simulate the seismic response of 2-D and 3-D geological structures, *Bull. Seism. Soc. Am.*, 88, 368-392, 1998.

Nguyen, N., Peraire, J., Cockburn, B., High-order implicit hybridizable discontinuous Galerkin methods for acoustics and elastodynamics, *J. Comp. Physics*, 230, 10, 3695-3718, 2011.

Wilcox, L., Stadler, G., Burstede, C., Ghattas, O., A high-order discontinuous Galerkin method for wave propagation through coupled elastic-acoustic media, *J. Comp. Phys.*, 229, 9379-9386, 2010.