

Tsunami Simulation using CIP Method with Characteristic Curve Equations and TVD-MacCormack Method

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After entering 21st century, we already had two big tsunami disasters associated with Mw9 earthquakes in Sumatra and Japan. To mitigate the damages of tsunami, the numerical simulation technology combined with information technologies could provide reliable predictions in planning countermeasures to prevent the damage to the social system, making safety maps, and submitting early evacuation information to the residents.

Shallow water equations are still solved not only for global scale simulation of the ocean tsunami propagation but also for local scale simulation of overland inundation in many tsunami simulators though three-dimensional model starts to be used due to improvement of CPU. One-dimensional shallow water equations are below:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = S \tag{1}$$

in which

$$Q = \begin{pmatrix} D \\ M \end{pmatrix},\tag{2}$$

$$E = \begin{pmatrix} M \\ M^2/D + gD^2/2 \end{pmatrix},$$
(3)

$$S = \begin{pmatrix} 0 \\ -gD\frac{\partial z}{\partial x} - gn^2 M |M| / D^{7/3} \end{pmatrix}.$$
(4)

where D[m] is total water depth; $M[m^2/s]$ is water flux; z[m] is topography; $g[m/s^2]$ is the gravitational acceleration; $n[s/m^{1/3}]$ is Manning's roughness coefficient.

To solve these, the staggered leapfrog scheme is used in a lot of wide-scale tsunami simulator. But this scheme has a problem that lagging phase error occurs when courant number is small. In some practical simulation, a kind of diffusion term is added. In this study, we developed two wide-scale tsunami simulators with different schemes and compared usual scheme and other schemes in practicability and validity. One is a total variation diminishing modification of the MacCormack method (TVD-MacCormack method) which is famous for the simulation of compressible fluids. The other is the Cubic Interpolated Profile (CIP) method with characteristic curve equations transformed from shallow water equations. Characteristic curve equations derived from shallow water equations are below:

$$\frac{\partial R_x^{\pm}}{\partial t} + C_x^{\pm} \frac{\partial R_x^{\pm}}{\partial x} = \mp \frac{g}{2} \frac{\partial z}{\partial x}$$
(5)

in which

$$R_x^{\pm} = \sqrt{gD} \pm \frac{u}{2},\tag{6}$$

$$C_x^{\pm} = u \pm \sqrt{gD}.\tag{7}$$

where u[m/s] is water velocity.

It is difficult to solve the inundation on the land with these methods though These two methods are applicable to the ocean tsunami propagation. We studied how to apply these methods to overland inundation and how to couple the ocean global model with the land local model.

Simple case studies of ocean tsunami propagation and overland tsunami inundation were performed to validate three methods comparing the results with theoretical solution. Finally, we performed case studies of the Great East Japan Earthquake in 2011 and confirmed the applicability to the actual tsunami.