



## **Topology of Sustainable Management of Dynamical Systems with Desirable States**

Jobst Heitzig (1) and Tim Kittel (1,2)

(1) Potsdam Institute for Climate Impact Research, Potsdam, Germany , (2) Humboldt-Universität zu Berlin, Department of Physics, Berlin, Germany

To keep the Earth System in a desirable region of its state space, such as the recently suggested “tolerable environment and development window”, “planetary boundaries”, or “safe (and just) operating space”, in addition to the identification of the quantitative internal dynamics and the available options for influencing it (management), there is an urgent need to understand the systems’ state space structure with regard to questions such as (i) which of its parts can be reached from which others with or without leaving the desirable region, (ii) which parts are in a variety of senses “safe” to stay in when management options break away, and which qualitative decision problems may occur as a consequence of this structure.

To complement existing approaches from optimal control focusing on quantitative optimization and being much applied in both engineering and integrated assessment, we develop a mathematical theory of the qualitative topology that partitions the state space of a dynamical system with management options and desirable states including terminology suggestions for the various resulting parts. Our detailed formal classification of the possible states and management options with respect to the possibility of avoiding or leaving the undesired region indicates that before performing some form of quantitative optimization, the sustainable management of the Earth System may require decisions of a more discrete type, e.g. choosing between ultimate safety and permanent desirability, or between permanent safety and increasing future options.