



The applications of Complexity Theory and Tsallis Non-extensive Statistics at Solar Plasma Dynamics

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As the solar plasma lives far from equilibrium it is an excellent laboratory for testing complexity theory and non-equilibrium statistical mechanics. In this study, we present the highlights of complexity theory and Tsallis non extensive statistical mechanics as concerns their applications at solar plasma dynamics, especially at sunspot, solar flare and solar wind phenomena. Generally, when a physical system is driven far from equilibrium states some novel characteristics can be observed related to the nonlinear character of dynamics. Generally, the nonlinearity in space plasma dynamics can generate intermittent turbulence with the typical characteristics of the anomalous diffusion process and strange topologies of stochastic space plasma fields (velocity and magnetic fields) caused by the strange dynamics and strange kinetics (Zaslavsky, 2002). In addition, according to Zelenyi and Milovanov (2004) the complex character of the space plasma system includes the existence of non-equilibrium (quasi)-stationary states (NESS) having the topology of a percolating fractal set. The stabilization of a system near the NESS is perceived as a transition into a turbulent state determined by self-organization processes. The long-range correlation effects manifest themselves as a strange non-Gaussian behavior of kinetic processes near the NESS plasma state. The complex character of space plasma can also be described by the non-extensive statistical thermodynamics pioneered by Tsallis, which offers a consistent and effective theoretical framework, based on a generalization of Boltzmann - Gibbs (BG) entropy, to describe far from equilibrium nonlinear complex dynamics (Tsallis, 2009). In a series of recent papers, the hypothesis of Tsallis non-extensive statistics in magnetosphere, sunspot dynamics, solar flares, solar wind and space plasma in general, was tested and verified (Karakatsanis et al., 2013; Pavlos et al., 2014; 2015). Our study includes the analysis of solar plasma time series at three cases: sunspot index, solar flare and solar wind data. The non-linear analysis of the sunspot index is embedded in the non-extensive statistical theory of Tsallis (1988; 2004; 2009). The q -triplet of Tsallis, as well as the correlation dimension and the Lyapunov exponent spectrum were estimated for the SVD components of the sunspot index timeseries. Also the multifractal scaling exponent spectrum $f(a)$, the generalized Renyi dimension spectrum $D(q)$ and the spectrum $J(p)$ of the structure function exponents were estimated experimentally and theoretically by using the q -entropy principle included in Tsallis non-extensive statistical theory, following Arimitsu and Arimitsu (2000, 2001). Our analysis showed clearly the following: (a) a phase transition process in the solar dynamics from high dimensional non-Gaussian SOC state to a low dimensional non-Gaussian chaotic state, (b) strong intermittent solar turbulence and anomalous (multifractal) diffusion solar process, which is strengthened as the solar dynamics makes a phase transition to low dimensional chaos in accordance to Ruzmaikin, Zelenyi and Milovanov's studies (Zelenyi and Milovanov, 1991; Milovanov and Zelenyi, 1993; Ruzmakin et al., 1996), (c) faithful agreement of Tsallis non-equilibrium statistical theory with the experimental estimations of: (i) non-Gaussian probability distribution function $P(x)$, (ii) multifractal scaling exponent spectrum $f(a)$ and generalized Renyi dimension spectrum D_q , (iii) exponent spectrum $J(p)$ of the structure functions estimated for the sunspot index and its underlying non equilibrium solar dynamics. Also, the q -triplet of Tsallis as well as the correlation dimension and the Lyapunov exponent spectrum were estimated for the singular value decomposition (SVD) components of the solar flares timeseries. Also the multifractal scaling exponent spectrum $f(a)$, the generalized Renyi dimension spectrum $D(q)$ and the spectrum $J(p)$ of the structure function exponents were estimated experimentally and theoretically by using the q -entropy principle included in Tsallis non-extensive statistical theory, following Arimitsu and Arimitsu (2000). Our analysis showed clearly the following: (a) a phase transition process in the solar flare dynamics from a high dimensional non-Gaussian self-organized critical (SOC) state to a low dimensional also non-Gaussian chaotic state, (b) strong intermittent solar corona turbulence and an anomalous (multifractal) diffusion solar corona process, which is strengthened as the solar corona dynamics makes a phase transition to low dimensional chaos, (c) faithful agreement of Tsallis non-equilibrium statistical theory with the experimental estimations of the functions: (i) non-Gaussian probability distribution function $P(x)$, (ii) $f(a)$ and $D(q)$, and (iii) $J(p)$ for the solar flares timeseries and its underlying non-equilibrium solar dynamics, and (d) the solar flare dynamical profile is revealed similar to the dynamical profile of the solar corona zone as far as the phase transition process from self-organized

criticality (SOC) to chaos state. However the solar low corona (solar flare) dynamical characteristics can be clearly discriminated from the dynamical characteristics of the solar convection zone. At last we present novel results revealing non-equilibrium phase transition processes in the solar wind plasma during a strong shock event, which can take place in Solar wind plasma system. The solar wind plasma as well as the entire solar plasma system is a typical case of stochastic spatiotemporal distribution of physical state variables such as force fields () and matter fields (particle and current densities or bulk plasma distributions). This study shows clearly the non-extensive and non-Gaussian character of the solar wind plasma and the existence of multi-scale strong correlations from the microscopic to the macroscopic level. It also underlines the inefficiency of classical magneto–hydro–dynamic (MHD) or plasma statistical theories, based on the classical central limit theorem (CLT), to explain the complexity of the solar wind dynamics, since these theories include smooth and differentiable spatial–temporal functions (MHD theory) or Gaussian statistics (Boltzmann–Maxwell statistical mechanics). On the contrary, the results of this study indicate the presence of non-Gaussian non-extensive statistics with heavy tails probability distribution functions, which are related to the q-extension of CLT. Finally, the results of this study can be understood in the framework of modern theoretical concepts such as non-extensive statistical mechanics (Tsallis, 2009), fractal topology (Zelenyi and Milovanov, 2004), turbulence theory (Frisch, 1996), strange dynamics (Zaslavsky, 2002), percolation theory (Milovanov, 1997), anomalous diffusion theory and anomalous transport theory (Milovanov, 2001), fractional dynamics (Tarasov, 2013) and non-equilibrium phase transition theory (Chang, 1992).

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