



Solution to the paradox of the linear stability of the Hagen–Poiseuille flow and the viscous dissipative mechanism of the arising of turbulence in a boundary layer

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It has been shown (Chefranov S.G. Chefranov A.G., JETP, 2014, 119(2), 331-340) that the conclusion of the linear instability of the Hagen–Poiseuille flow at finite Reynolds numbers requires the refusal of the use of the traditional “normal” form of the representation of disturbances, which implies the possibility of separation of variables describing disturbances as functions of the radial and longitudinal (along the axis of a tube) coordinates. In the absence of such separation of variables in the developed linear theory, it has been proposed to use a modification of the Bubnov–Galerkin theory that makes it possible to take into account the difference between the periods of the longitudinal variability for different radial modes preliminarily determined by the standard application of the Galerkin–Kantorovich method to the evolution equation of extremely small axisymmetric disturbances of the tangential component of the velocity field. It has been shown that the consideration of even two linearly interacting radial modes for the Hagen–Poiseuille flow can provide linear instability only in the presence of the mentioned conditionally periodic longitudinal variability of disturbances along the axis of the tube, when the threshold Reynolds number $R_{th}(p)$ is very sensitive to the ratio p of two longitudinal periods each describing longitudinal variability for its radial disturbance mode. In this case, the threshold Reynolds number can tend to infinity only at $p = p_k = k$, $p = p_{1/k} = 1/k$, and $p = p_{k0.5} = [k + 1 \pm ((k+1)^2 - 4)^{0.5}] / 2$, where $k = 1, 2, 3, \dots$. The minimum Reynolds number $R_{th}(p) = 448$ (at which $p = 1.527$) for the linear instability of the Hagen–Poiseuille flow quantitatively corresponds to the condition of the excitation of Tollmien–Schlichting waves in the boundary layer, where $R_{th} = 420$. Similarity of the mechanisms of linear viscous dissipative instability for the Hagen–Poiseuille flow and Tollmien–Schlichting waves has been discussed. Good quantitative agreement has been obtained between the phase velocities of the vortex disturbances and the experimental data on the velocities of the leading and trailing edges of turbulent “puffs” propagating along the axis of the tube.