



Reactive-infiltration instability in radial geometry

Piotr Grodzki and Piotr Szymczak

Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warszawa, Poland

A planar dissolution front propagating through a homogeneous porous matrix is unstable with respect to small variations in local permeability; regions of high permeability dissolve faster because of enhanced transport of reactants, which leads to increased rippling of the front. This phenomenon, usually referred to as reactive-infiltration instability is an important mechanism for pattern development in geology, with a range of morphologies and scales, from cave systems running for hundreds of miles to laboratory acidization on the scale of centimeters.

In general, this instability is characterized by two length scales: the diffusive length (D/v) and the reactant penetration length (v/r), where v is the Darcy velocity, D - the diffusion constant and r - the dissolution rate. If the latter scale is much smaller than the former one can adopt the so-called thin front limit, where the interface is treated as a discontinuity in porosity, with a completely dissolved phase on one side and an undissolved phase on the other. Linear stability analysis for this case has been carried out by Chadam et al. [1], and the corresponding dispersion relation shows that long wavelengths are unstable, whereas short wavelengths are stabilized by diffusion.

In their derivation, Chadam et al. have considered a linear geometry with a uniform pressure gradient applied along one of the directions. However, in many cases (e.g. in the acidization techniques used in oil industry) the reactive fluids are injected through a well and thus the relevant geometry is radial rather than linear. Motivated by this, we have carried out the linear stability analysis of the reactive-infiltration problem in radial geometry, with the fluid injection at the centre of the system. We stay within the thin-front limit and derive the corresponding dispersion relation, which shows the stable regions for both the long-wavelength and short-wavelength modes, and the unstable region in between. Next, we study how the instability growth rate depends on the Peclet number (Pe) and permeability contrast between the undissolved and dissolved phase (Γ) and find the region in the (Pe, Γ) space when the system is absolutely stable. This behaviour is in contrast to the viscous fingering problem in radial geometry [2], where for a given flow rate the front always becomes eventually unstable, after reaching a certain critical radius R .

[1] J. Chadam, D. Ho, E. Merino, P. Ortoleva, A. Sen, Reactive Infiltration Instabilities, *IMA J. Appl. Math.* 36, 207-221 (1986)

[2] L. Paterson, Radial fingering in a Hele Shaw cell, *J. Fluid Mech.* 113, 513-529 (1981)