



## Towards a unified solution of localization failure with mixed finite elements

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Notwithstanding computational scientists made significant steps in the numerical simulation of failure in last three decades, the strain localization problem is still an open question. Especially in a geotechnical setting, when dealing with stability analysis of slopes, it is necessary to provide correct distribution of displacements, to evaluate the stresses in the ground and, therefore, to be able to identify the slip lines that brings to progressive collapse of the slope.

Finite elements are an attractive method of solution thanks to profound mathematical foundations and the possibility of describing generic geometries. In order to account for the onset of localization band, the smeared crack approach [1] is introduced, that is the strain localization is assumed to occur in a band of finite width where the displacements are continuous and the strains are discontinuous but bounded.

It is well known that this kind of approach poses some challenges. The standard irreducible formulation of FEM is known to be heavily affected by spurious mesh dependence when softening behavior occurs and, consequently, slip lines evolution is biased by the orientation of the mesh. Moreover, in the case of isochoric behavior, unbounded pressure oscillations arise and the consequent locking of the stresses pollutes the numerical solution. Both problems can be shown not to be related to the mathematical statement of the continuous problem but instead to its discrete (FEM) counterpart.

Mixed finite element formulations represent a suitable alternative to mitigate these drawbacks. As it has been shown in previous works by Cervera [2], a mixed formulation in terms of displacements and pressure not only provides a propitious solution to the problem of incompressibility, but also it was found to possess the needed robustness in case of strain concentration.

This presentation introduces a (stabilized) mixed finite element formulation with continuous linear strain and displacement interpolations. As a fundamental enhancement of the displacement-pressure formulation above mentioned, this kind of formulation benefits of the following advantages: it provides enhanced rate of convergence for the strain (and stress) and it is able to deal with incompressible situations. The method is completed with constitutive laws from Von Mises and Drucker-Prager local plasticity models with nonlinear strain softening. Moreover, global and local error norms are discussed to support the advantages of the proposed method.

Then, numerical examples of stability analysis of slopes are presented to demonstrate the capability of the method. It will be shown that not only soil slopes can be modeled but also snow avalanche release and their weak layer fracture can be similarly treated. Consequently, this formulation appears to be a general and accurate tool for the solution of mechanical problem involving failure with localization bands [3,4].

### References

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