



Least-squares-based adjustment of geodetic measurements with multiplicative random errors

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Adjustment has been based on the assumption that random errors of measurements are added to functional models, which are also called additive error models. However, modern earth-space observation technology has clearly demonstrated that measurement errors can be proportional to the true values of measurements such as GPS, VLBI baselines and LiDAR. Observational models of this type are called multiplicative error models. For example, in geodetic practice, we know that accuracy formulae of modern geodetic measurements often consist of two parts: one proportional to the measured quantity and the other constant. From the statistical point of view, such measurements are of mixed multiplicative and additive random errors. Nevertheless, almost no adjustment has been developed to strictly address geodetic data contaminated by mixed multiplicative and additive random errors from the statistical point of view. We systematically develop adjustment methods for geodetic data contaminated with multiplicative and additive errors. More precisely, we discuss the ordinary least squares (LS) and weighted LS methods and extend the bias-corrected weighted LS method of Xu and Shimada (Commun Stat B29:83–96, 2000) to the case of mixed multiplicative and additive random errors. We derive the biases of weighted LS estimates. We analytically derive the variance-covariance matrices of the three LS-based adjustments, the adjusted measurements and the corrections of measurements in multiplicative error models. For quality evaluation, we construct five estimators for the variance of unit weight in association of the three LS-based adjustment methods, which are then further compared, both theoretically and by simulations. The three LS-based methods are then demonstrated and compared with a synthetic example of surface interpolation. The bias-corrected weighted LS estimate is unbiased up to the second order approximation and is of the best accuracy. Although the ordinary LS method can warrant an unbiased estimate for a linear model with multiplicative and additive errors, it is less accurate and always produces a very poor estimate of the variance of unit weight, if we follow the estimate valid for additive error models.