Geophysical Research Abstracts
Vol. 17, EGU2015-7942, 2015
EGU General Assembly 2015

Joint inversion : Exploring the different ways of coupling geophysical and groundwater data

Klara Steklova (1) and Eldad Haber (1,2)

(1) Earth and Ocean Sciences, The University of British Columbia, Vancouver, BC, Canada (ksteklova@eos.ubc.ca), (2) Mathematics, The University of British Columbia, Vancouver, BC, Canada (ehaber@eos.ubc.ca)

Hydrogeophysical studies combining different data types to improve the estimates of hydrological states have recently gained much attention. One of the typical hydrogeophysical applications is solute transport, such as the case of seawater intrusions. Here the freshwater and seawater exhibit different electrical conductivities, making, for example electromagnetic geophysical data collection ideal addition to the standard groundwater well data. However, the actual methods for coupling the groundwater and geophysical variables, as well as integrating the two different models through inversion, are still a subject of research.

Groundwater models are usually highly parameterized with arbitrary scales and sparse observed data compared to geophysical models which tend to have a large but indirect set of data, and a scale and resolution dependent on the chosen method. Solving a coupled inverse problem to estimate the hydrological states, such as solute distribution, then faces two main challenges: determining the relationship between geophysical and groundwater variables; and fitting two different sources of data.

To invert both the geophysical and hydrological data sets we formulate the following objective function for electrical conductivity σ, and groundwater variable, solute fraction ω:

$$\begin{aligned} \min_{\sigma,\omega} \quad & \frac{1}{2}\|d_e - Q_e u(\sigma)\|^2_{\Sigma_e^{-1}} + \frac{1}{2}\|d_f - Q_f\omega\|^2_{\Sigma_f^{-1}} + \beta_e R(\sigma) + \beta_f R(\omega) \\ \text{s.t} \quad & \sigma = p(\omega) \end{aligned} \tag{1}$$

where, d_e and d_f are geophysical and groundwater data, $Q_{e,f}$ data projection matrices, u is the measured electrical potential (dependent on σ), R is the regularization operator (e.g. L_2 norm) and β_e and β_f are regularization weights. In this case it is assumed that the petrophysical relationship $p(\sigma)$ between σ and ω is known, and is thus set as a constraint.

To solve the coupled problem we investigate two approaches, the first one is using the alternate direction method of multipliers (ADMM). The augmented Lagrangian for ADMM is then given by,

$$\begin{aligned} \mathcal{L}(\sigma,\omega,y) = \quad & \frac{1}{2}\|d_e - Q_e u(\sigma)\|^2_{\Sigma_e^{-1}} + \frac{1}{2}\|d_f - Q_f\omega\|^2_{\Sigma_f^{-1}} + \beta_e R(\sigma) + \beta_f R(\omega) \\ & + y^\top(\sigma - p(\omega)) + \frac{\rho}{2}\|\sigma - p(\omega))\|^2, \end{aligned} \tag{2}$$

where y is the Lagrange multiplier and ρ is a parameter that can be chosen somewhat arbitrarily. At each iteration $\mathcal{L}(\sigma,\omega,y)$ is minimized with respect to σ or ω and y is updated. This method provides a huge computational advantage since at each iteration we solve only a subproblem with one data misfit term, regularization term, and coupling terms where one of the variables is fixed. However, all the involved terms need to be differentiable in order to proceed with a Gauss - Newton type minimization method.

The second approach can be followed if the empirical relationship between ω and σ is unknown. In this case, the unknown relationship is replaced by some structure similarity mapping, e.g. joint total variation (JTV),

$$JTV(\sigma,\omega) = \int \sqrt{|\nabla\sigma|^2 + |\nabla\omega|^2}ds.$$

JTV is differential w.r.t both σ and ω and has also advantage of being convex. The objective function (Eq.1) then contains additional JTV term instead of the constraint and can be minimized by block coordinate descent method.

Both geophysical and groundwater models were developed in Matlab, including sensitivities of data w.r.t σ and ω based on a discretized system of equations. The joint inversion outlined above was tested on the synthetic case of seawater intrusion and a solute tracer test with promising results.