Comparison between two formulae for parameter evaluation based on the ensemble Kalman filter

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The ensemble Kalman filter is regarded as a sequential algorithm based on the state space modeling. When the state space model is employed, unknown model parameters can be estimated using the marginal likelihood, which is a metric of fitness to a sequence of observations. However, we can consider two different ways of approximations in computing the marginal likelihood based on the ensemble Kalman filter. One is the Gaussian approach and the other is the Monte Carlo approach. In the Gaussian approach, only the first and second order moments of the ensemble are taken into consideration, and the marginal likelihood is computed on the basis of a Gaussian approximation. In the Monte Carlo approximation, the ensemble is regarded as a set of samples representing a non-Gaussian probability density function, and the marginal likelihood is computed by Monte Carlo integration. These two formulae do not agree with each other even in cases of linear Gaussian problems, and they provide different values in general. In this study, the performance of the parameter estimation are compared between these two different approaches. It was confirmed that both approaches yield some random errors due to inherent randomness in the ensemble Kalman filter. However, the Gaussian approach tended to provide better marginal likelihood values in the case of linear Gaussian cases. In addition, the results of experiments with linear Gaussian problems showed that estimates obtained by the Monte Carlo approach tended to be biased even if the system and observation are linear and Gaussian. The possible reason of the bias and some potential improvements to calculate the marginal likelihood for nonlinear and non-Gaussian cases will be discussed.