



Pipelined Flexible Krylov Subspace Methods

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State-of-the-art geophysical forward models expend most of their computational resources solving large, sparse linear systems. To date, preconditioned Krylov subspace methods have proven to be the only algorithmically scalable approach to solving these systems. However, at 'extreme scale', the global reductions required by the inner products within these algorithms become a computational bottleneck, and it becomes advantageous to use pipelined Krylov subspace methods. These allow overlap of global reductions with other work, at the expense of using more storage and local computational effort, including overhead required to synchronize overlapping work. An impediment to using currently-available pipelined solvers for relevant geophysical forward modeling is that they are not 'flexible', meaning that they cannot support nonlinear or varying preconditioners. Such preconditioners are effective for solving challenging linear systems, notably those arising from modelling of Stokes flow with highly heterogeneous viscosity structure. To this end, we introduce, for the first time, Krylov subspace methods which are both pipelined and flexible. We implement and demonstrate pipelined, flexible Conjugate Gradient, GMRES, and Conjugate Residual methods, which will be made publicly available via the open source PETSc library. Our algorithms are nontrivial modifications of the flexible methods they are based on (that is, they are not equivalent in exact arithmetic), so we analyze them mathematically and through a number of numerical experiments employing multi-level preconditioners. We highlight the benefits of these algorithms by solving variable viscosity Stokes problems directly relevant to lithospheric dynamics.