



## **Straight line fitting and predictions: On a marginal likelihood approach to linear regression and errors-in-variables models**

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Linear regression methods are without doubt the most used approaches to describe and predict data in the physical sciences. They are often good first order approximations and they are in general easier to apply and interpret than more advanced methods. However, even the properties of univariate regression can lead to debate over the appropriateness of various models as witnessed by the recent discussion about climate reconstruction methods.

Before linear regression is applied important choices have to be made regarding the origins of the noise terms and regarding which of the two variables under consideration that should be treated as the independent variable. These decisions are often not easy to make but they may have a considerable impact on the results. We seek to give a unified probabilistic - Bayesian with flat priors - treatment of univariate linear regression and prediction by taking, as starting point, the general errors-in-variables model (Christiansen, *J. Clim.*, 27, 2014-2031, 2014). Other versions of linear regression can be obtained as limits of this model.

We derive the likelihood of the model parameters and predictands of the general errors-in-variables model by marginalizing over the nuisance parameters. The resulting likelihood is relatively simple and easy to analyze and calculate. The well known unidentifiability of the errors-in-variables model is manifested as the absence of a well-defined maximum in the likelihood. However, this does not mean that probabilistic inference can not be made; the marginal likelihoods of model parameters and the predictands have, in general, well-defined maxima. We also include a probabilistic version of classical calibration and show how it is related to the errors-in-variables model. The results are illustrated by an example from the coupling between the lower stratosphere and the troposphere in the Northern Hemisphere winter.