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Application of Chance-Constrained Stochastic Optimization for Mitigating Downstream Flood Risks

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Motivation (1)

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Short-term Reservoir Optimization by Stochastic Optimization for Mitigation Downstream Flood Risks

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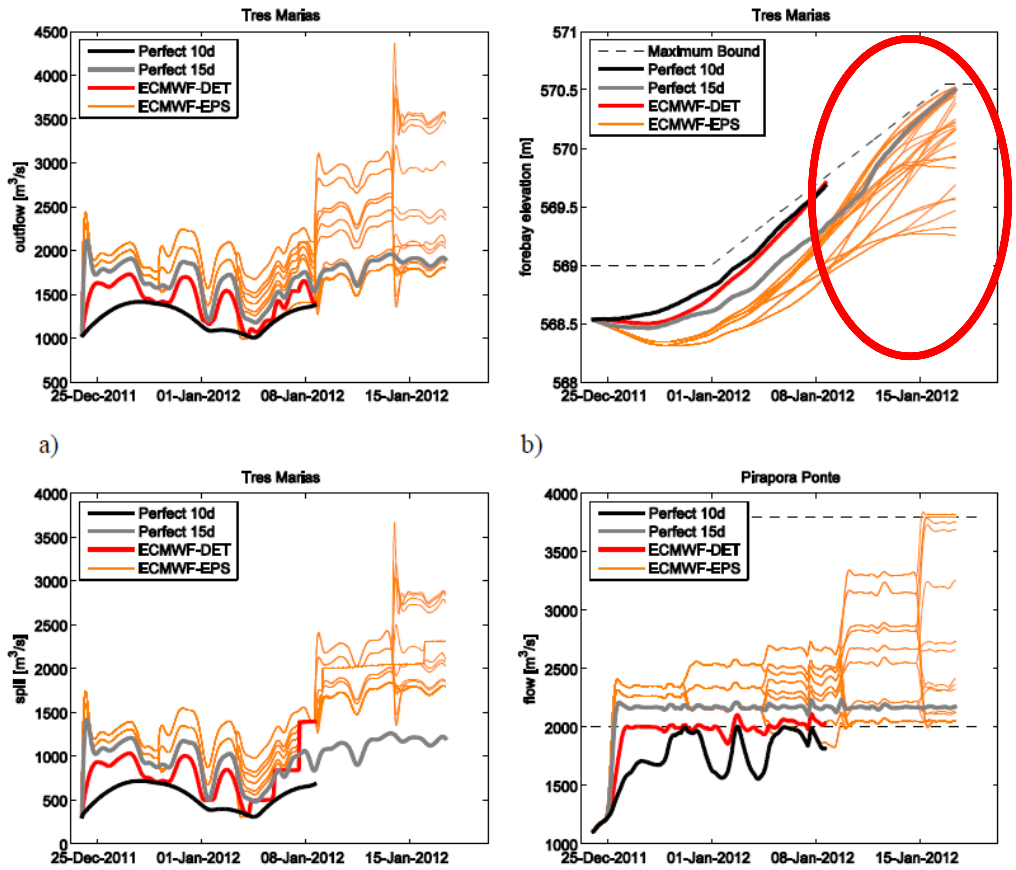
An important objective of the operation of multi-purpose reservoirs is the mitigation of flood risks in downstream river reaches. Under the assumptions of reservoirs with finite storage capacities, a key factor for its effective use during flood events is the proper timing of detention measures under consideration of forecast uncertainty. Operational flow forecasting systems support this task by providing deterministic or probabilistic inflow forecasts and decision support components for assessing optimum release strategies.

We focus on the decision support component and propose a deterministic optimization and its extension to stochastic optimization procedures based on the non-adaptive Sample Average Approximation (SAA) approach and an adaptive multi-stage stochastic optimization with underlying scenario trees. These techniques are used to compute release trajectories of the reservoirs over a finite forecast horizon of up to 14 days by integrating a nonlinear gradient-based optimization algorithm and a model of the water system. The latter consists of simulation components for pool routing and kinematic or diffusive wave models for the downstream river reaches including a simulation mode and a reverse adjoint mode for the efficient computation of first-order derivatives.

The framework has been implemented for a reservoir system operated by the Brazilian Companhia Energética de Minas Gerais S.A. (CEMIG). We present results obtained for the operation of the Três Marias reservoir in the Brazilian state of Minas Gerais with a catchment area of near 55,000 km2, an installed capacity of 396 MW and operation restrictions due to downstream flood risk. The focus of our discussion is the impact of sparsely available ground data, forecast uncertainty and its consideration in the optimization procedure. We compare the performance of the above mentioned optimization techniques and conclude the superiority of the stochastic methods.



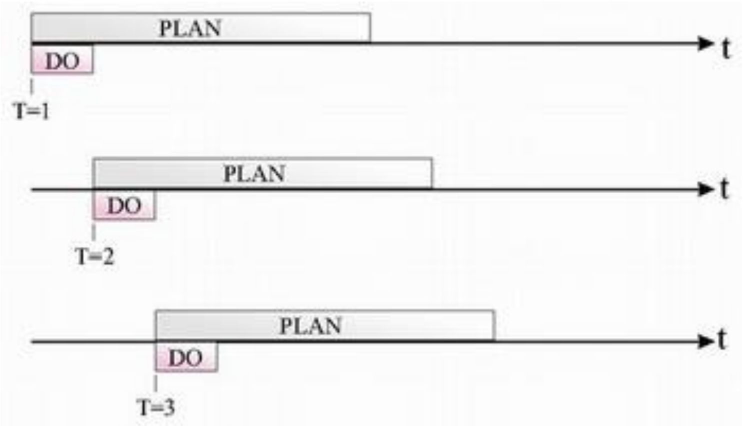
Stochastic optimization showed a more conservative allocation of storage due to the forebay elevation hard constraint. The control trajectory of the stochastic optimization meets the constraint for all 32 branches of the scenario tree.

Introduction – Model Predictive Control

Model Predictive Control is an online optimization approach originally developed by control engineers:

- predicts future system states over a forecast horizon by an internal model,
- expresses control performance mathematically in a cost function,
- minimize this cost function by an optimization algorithm,
- apply first steps of the optimum control trajectory and repeats the procedure when new information becomes available (receding horizon principle)

The control becomes anticipatory.



Source: Institut für Automatic, ETH, Zurich

Methodology (1)

In MPC the following system is considered:

states → $x^k = f(x^{k-1}, x^k, u^k, d^k)$ ← control, disturbance

dependent variable → $y^k = g(x^k, u^k, d^k)$

f, g → Water resources model

A deterministic optimization problem setup is given by:

$$\min_{u, x^* \in \{0,T\}} \sum_{k=1}^{N} J\left(x^k, y^k, u^k\right) + E\left(x^N, y^N, u^N\right)$$ ⇐ Objective function + soft constraints

subject to: $h(x^{*,k}, y^k, u^k, d^k) \leq 0, \quad k = 1, \ldots, N$ ⇐ Hard constraints

$$x^{*,k} - f(x^{*,k-1}, x^{*,k}, u^k, d^k) = 0$$ ⇐ Process model (simultaneous setup)

Methodology (2)

The stochastic approach considers an extension of the deterministic optimization by using the following objective function:

$$\min_{u,x^* \in \{0..T\}} \sum_{j=1}^{\overbrace{m}^{\text{number of scenarios}}} \sum_{k=1}^{n} \underbrace{p_j}_{\text{probablity of each scenario}} [J_j(x^{j,k}, y^{j,k}, u^{\mathbf{M}(j,k)}) + E_j(x^{j,N}, y^{j,N}, u^{\mathbf{M}(j,N)})]$$

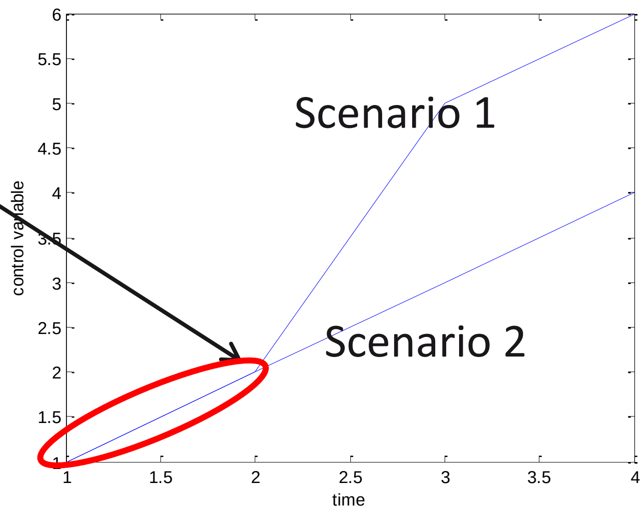
Matrix M assigns the control at time step k of scenario j to the control vector u. It becomes a practical way for defining the scenario tree nodal partition.

Example:

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 5 & 6 \end{bmatrix}$$

Branching point

Common control trajectory



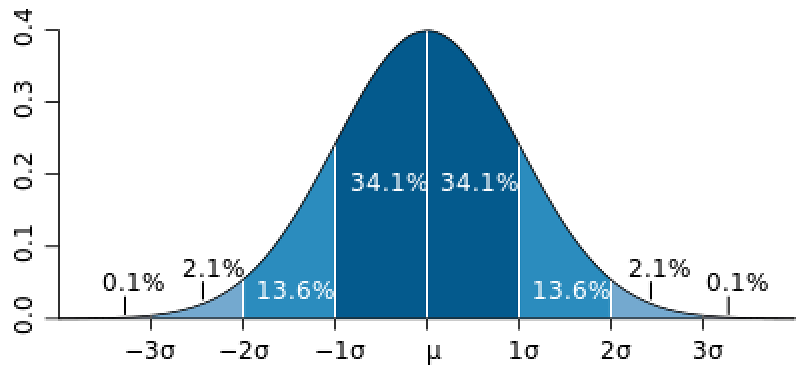
Methodology (3)

Chance constraints can be introduced through the following formulation into the objective functions.

- We assume a normal distribution of variable *v* (either state or dependent variable) and estimate *μ* and *σ* as:

$$\mu^k = \sum_{j=1}^{m} \left(p_j \cdot v^{j,k} \right)$$

$$\sigma^k = \sqrt{\sum_{j=1}^{m} \left(p_j \cdot \left(v^{j,k} - \mu \right)^2 \right) + \varepsilon}$$



Then:

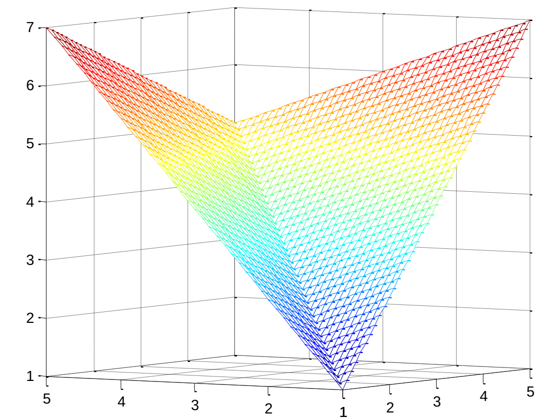
$$\boxed{\begin{aligned} & v_{\min}^k \le \mu^k \pm f \cdot \sigma^k \le v_{\max}^k \\ & \mu^k + f \cdot \sigma^k - v_{\max}^k \le 0 \\ & 0 \le \mu^k - f \cdot \sigma^k - v_{\min}^k \end{aligned}}$$

$$J += \left(\mu \pm f \cdot \sigma - v_{\min,\max} \right)^{ord}$$

Methodology (4)

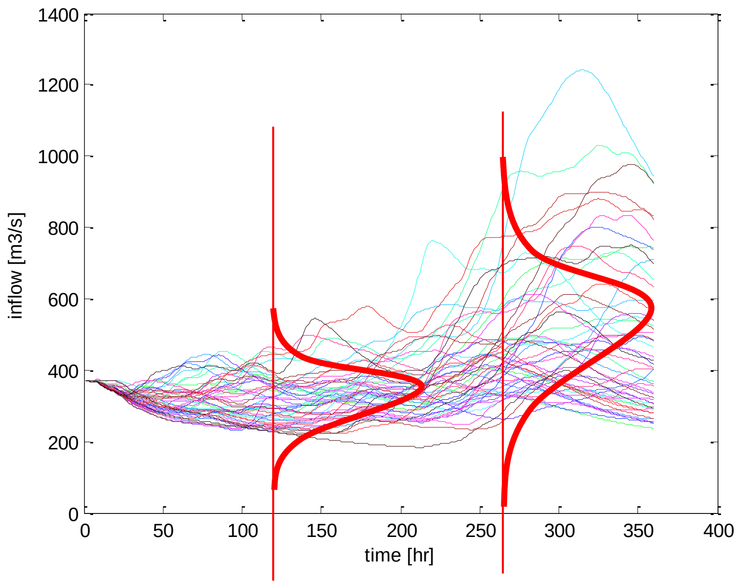
Relevance of ε for adjoint model. The partial derivatives can be computed using:

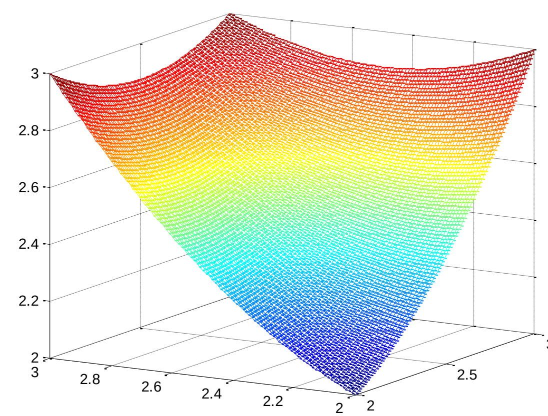
$$\frac{\partial J}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\mu + f\cdot\sigma - x_{\max}\right)^n = n\cdot\left(\mu + f\cdot\sigma - x_{\max}\right)^{n-1}\cdot\left[\frac{\partial \mu}{\partial x_i} + f\,\frac{\partial \sigma}{\partial x_i}\right]$$



From where it expands to:

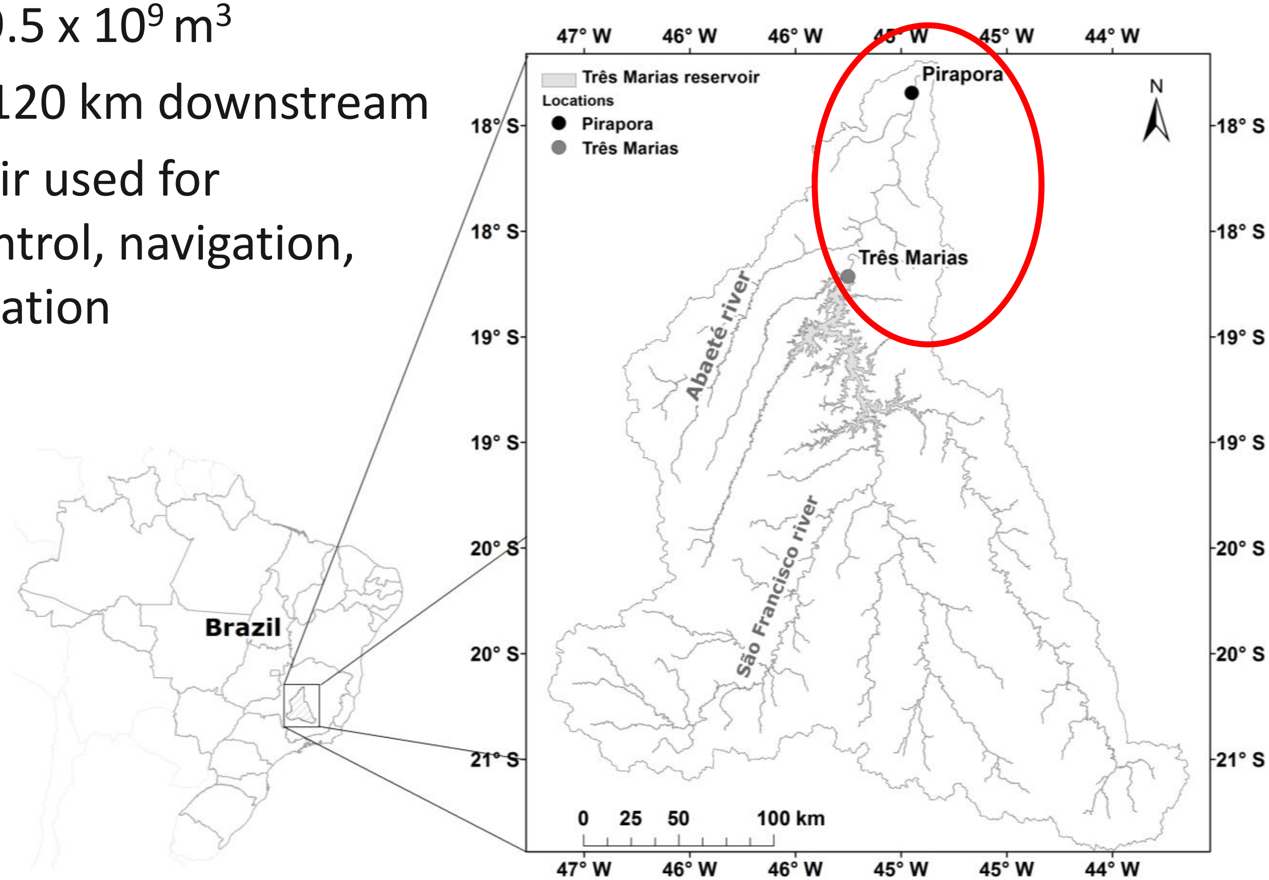
$$\frac{\partial J}{\partial x_i} = n\cdot\left(\mu + f\cdot\sigma - x_{\max}\right)^{n-1}\cdot\left[p_i + f\,\sigma^{-1}\,p_i\cdot\left(x_i - \mu\right)\right]$$





Case: Três Marias reservoir, Brazil

- Basin located in Minas Gerais state
- Catchment area: 55 000 km^2
- Reservoir capacity: 19.5 x 10^9 m^3
- Pirapora city located 120 km downstream
- Multipurpose reservoir used for hydropower, flood control, navigation, water supply and irrigation



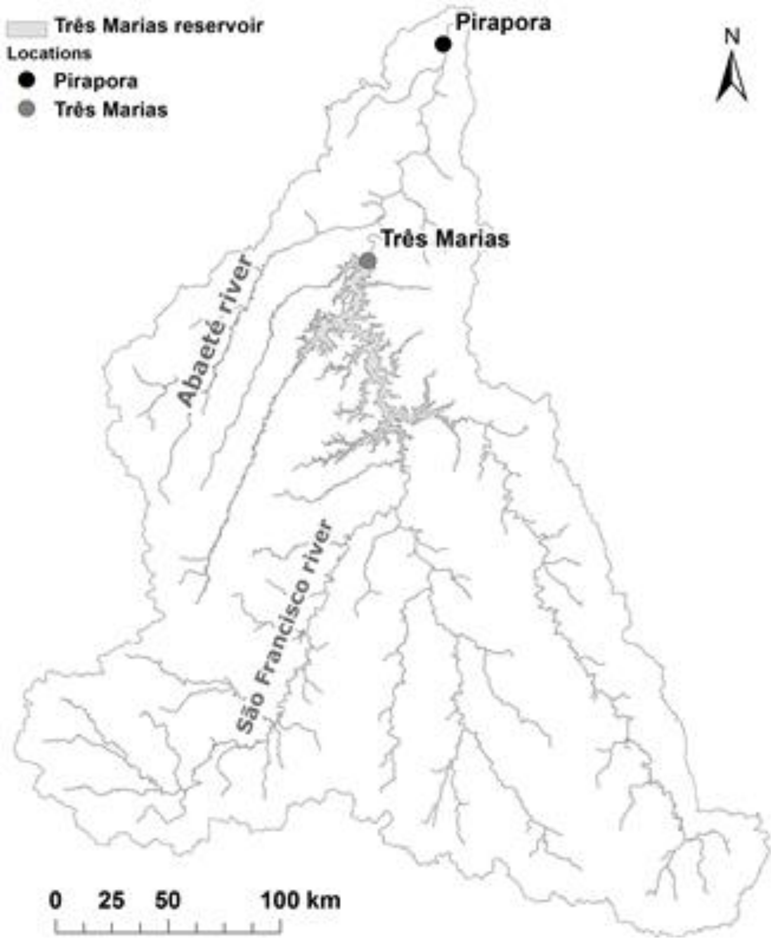
Case: Três Marias reservoir, Brazil

- Ensemble data from ECMWF every 12 hours
- Time steps of 6 hours
- Downscaled and disaggregated to hourly time steps
- Use of MGB-IPH model for streamflow forecasting (distributed model)

Case: Três Marias reservoir, Brazil

Optimization requirements:

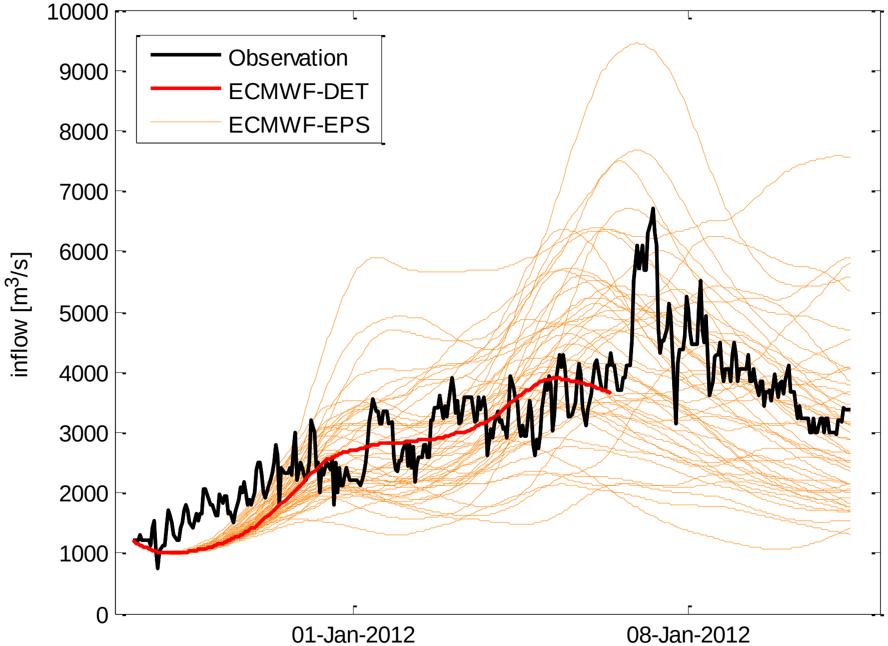
- Spill is undesired and it is linearly penalized (inside objective function).
- Spill is motivated by a time dependent maximum forebay elevation
- Two flow thresholds are given at Pirapora: 2000 and 3800 m3/s.
- Both correspond to soft constraints, although the second one has a much higher weighting coefficient in the objective function.
- A rate-of-change penalty is used for reservoir outflow to smooth the solution anc avoid high outflow gradients.



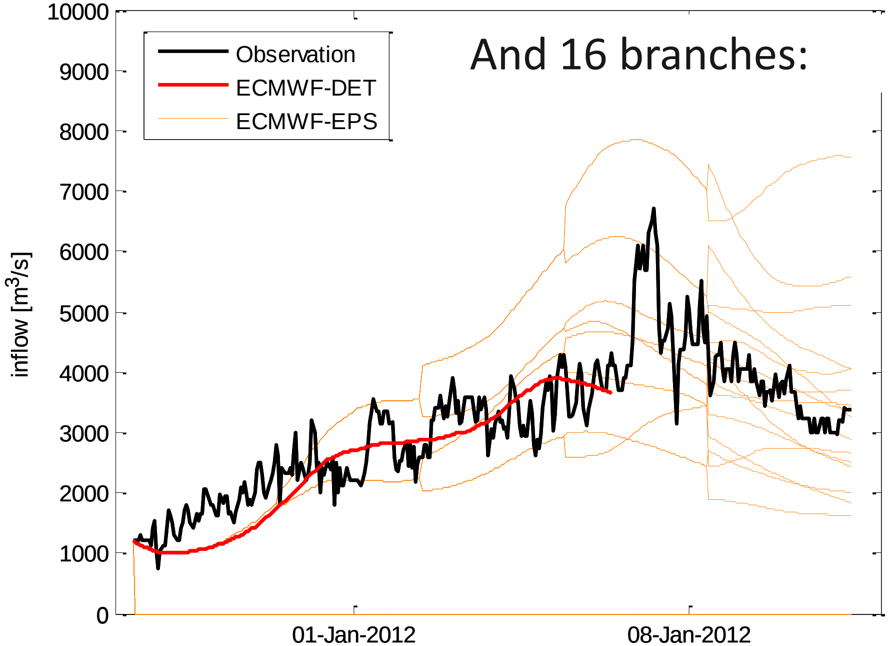
Results (1) – event on December 27, 2011

- Quantitative Precipitation Forecasts (QPF) from ECMWF Ensemble Prediction System (EPS) are used in combination with MGB-IPH model to produce streamflow forecasts
- Model is calibrated using data in the period from December 2006 to June 2011

Ensemble inflow prediction and related binary scenario tree with 32 branches



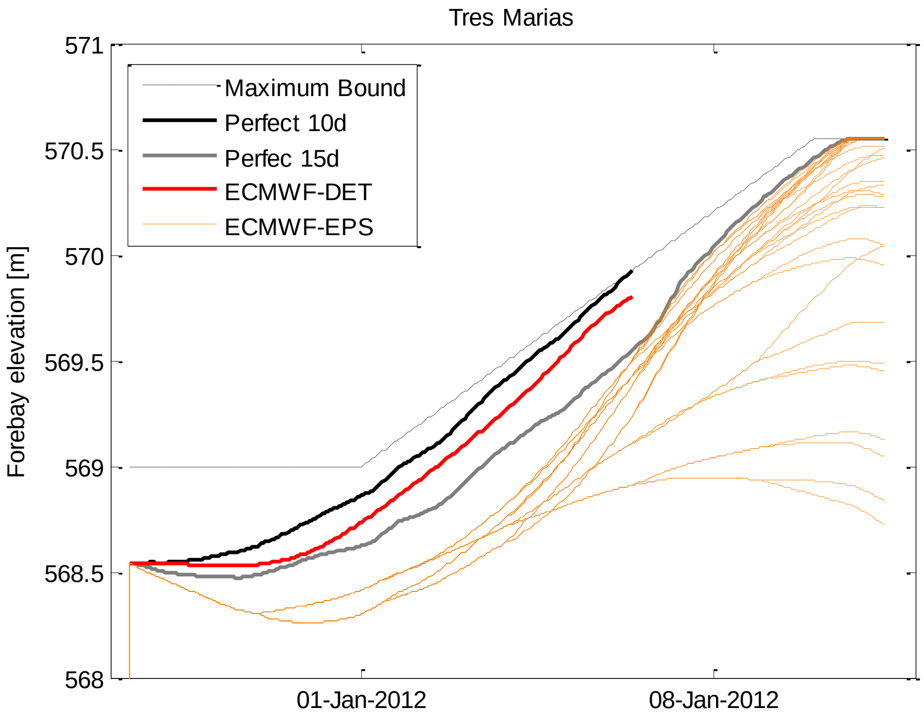
And 16 branches:

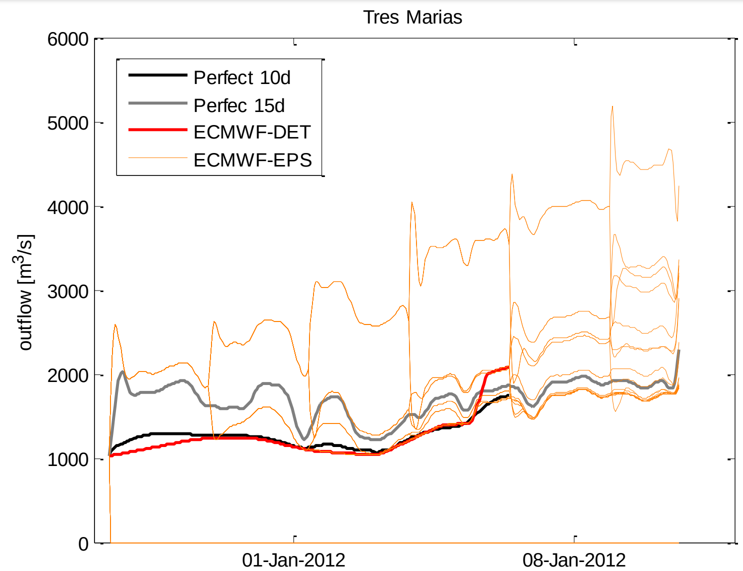


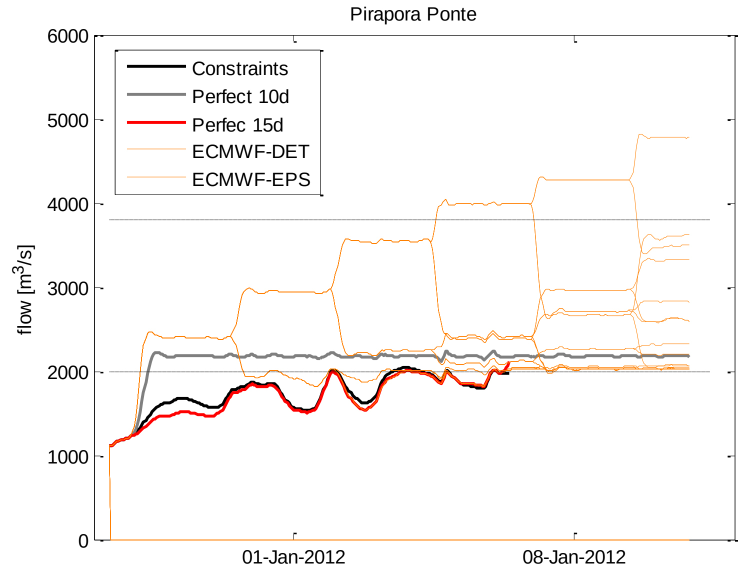
Results (2) – event on December 27, 2011

Without chance constraints

32 members



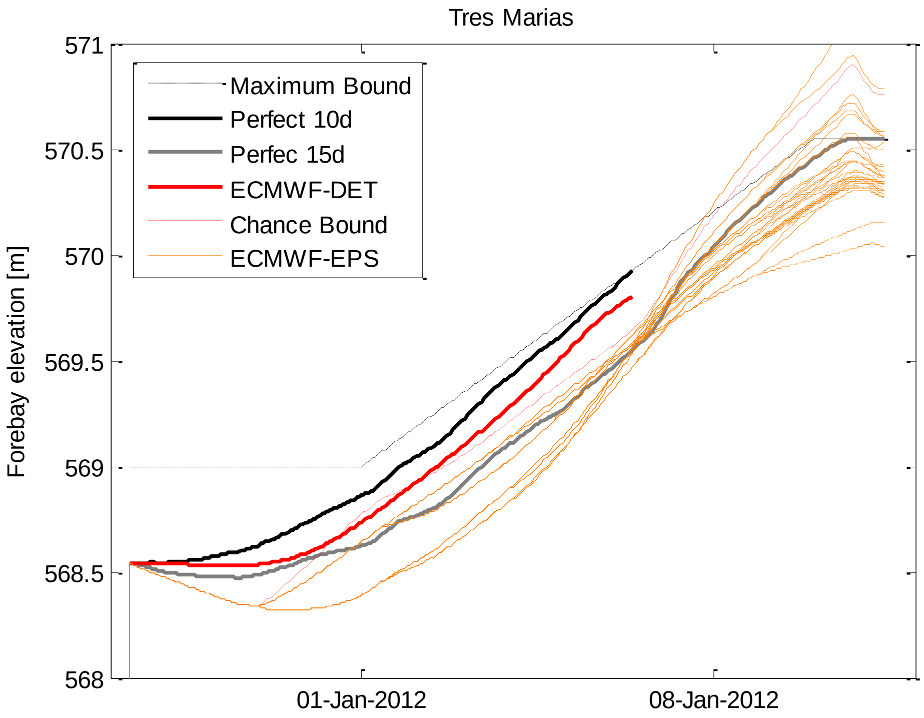


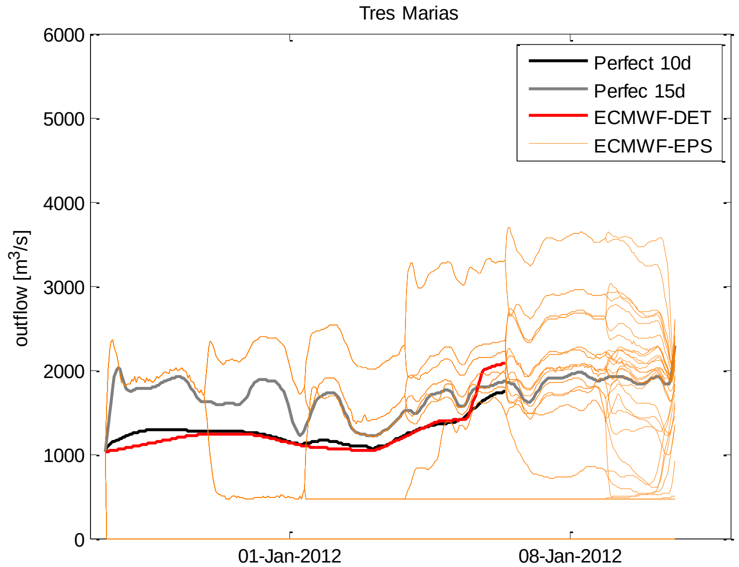


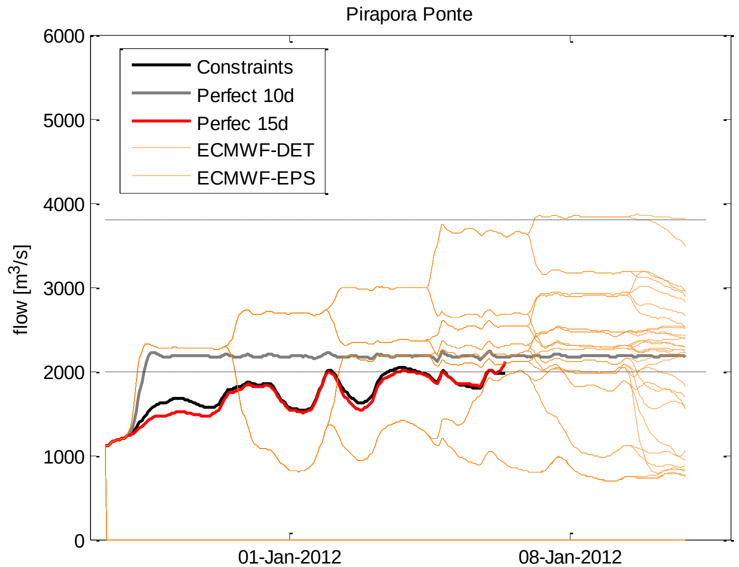
Results (3) – event on December 27, 2011

With chance constraints

32 members



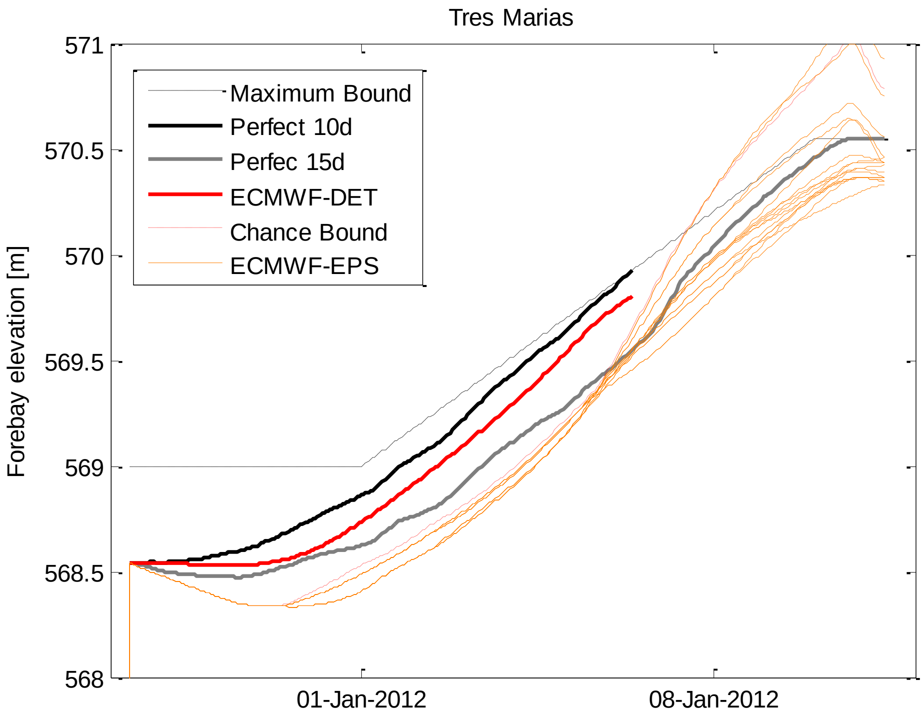


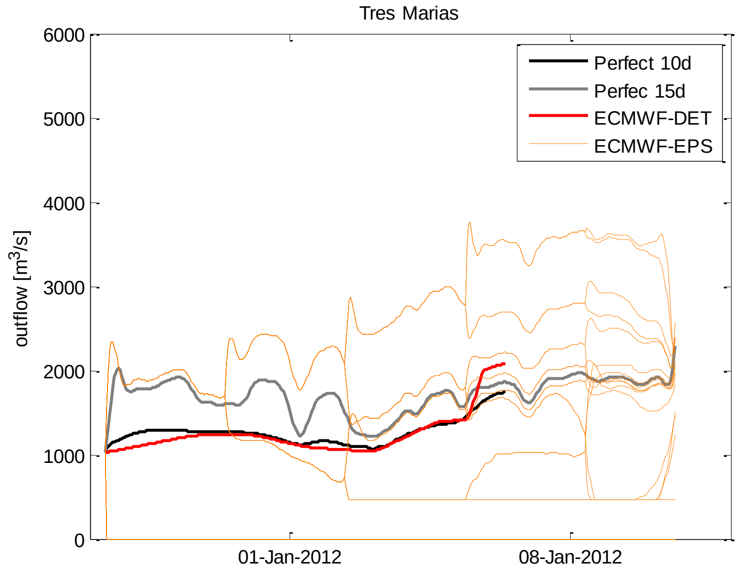


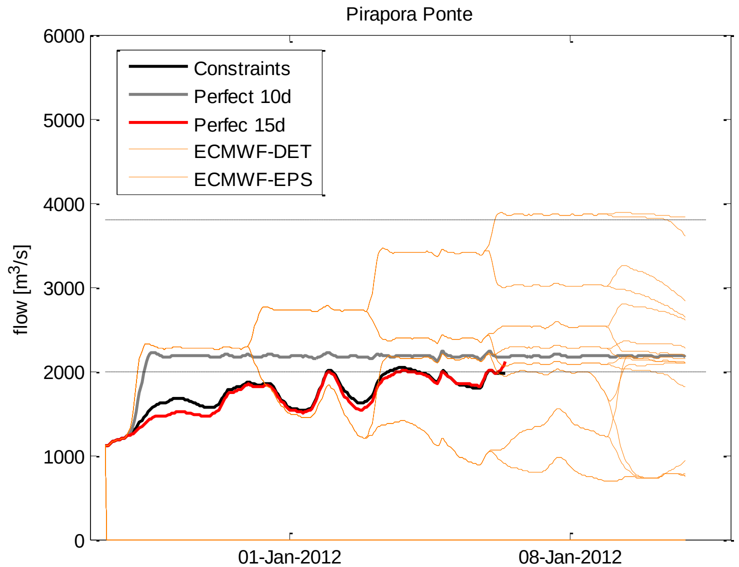
Results (4) – event on December 27, 2011

With chance constraints

16 members

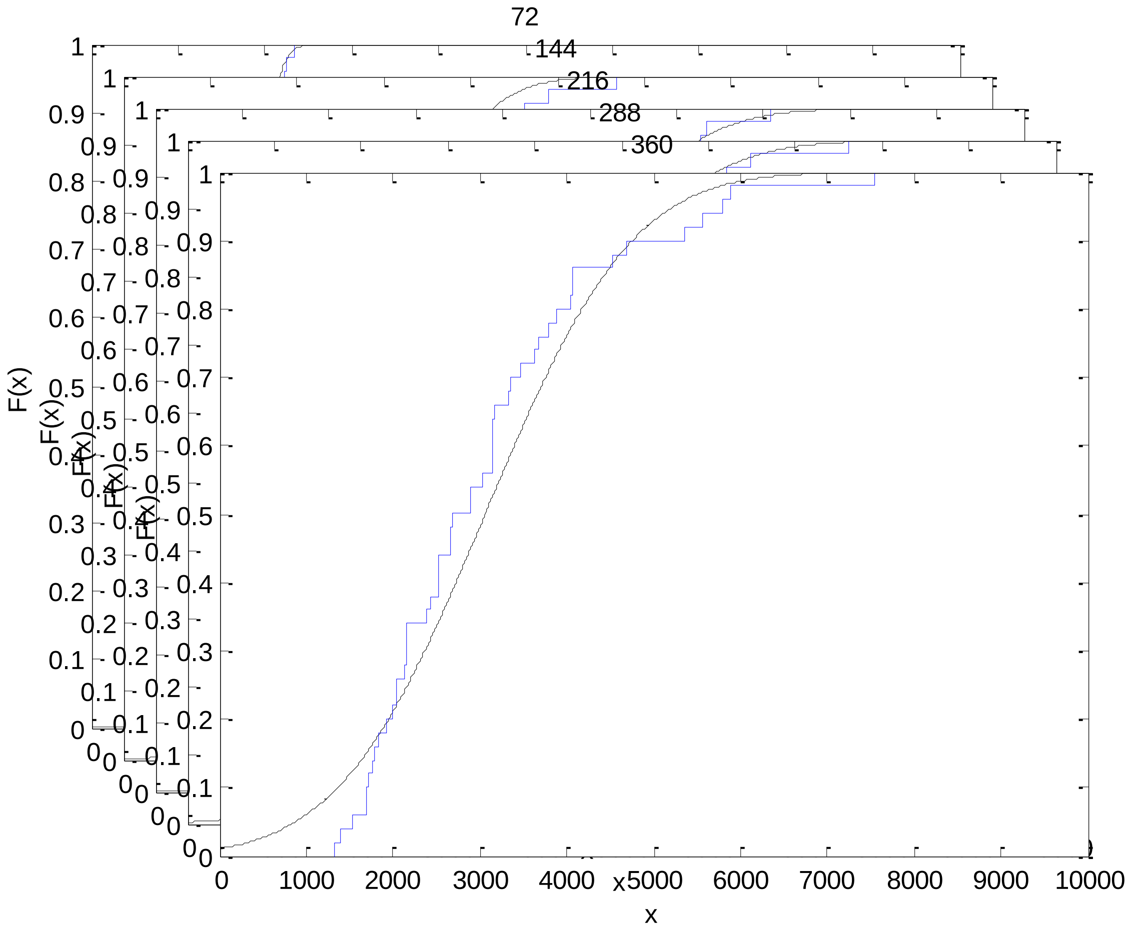






Results (2) – event on December 27, 2011

Cumulative distribution function (CDF) for inflows to Tres Marias reservoir using 50 members; plots every 3 days



Conclusions

- Typical inequality constraints require a full compliance of each ensemble's trajectory and neglect the uncertainty distribution associated to the ensemble.
- More flexible approach that <u>does not depend on the number of ensembles but rather on the distribution of uncertainty</u>.
- The extension of the ensemble leads to a more accurate representation of uncertainty. The <u>control trajectory is not necessarily modified by a larger ensemble, as long as the statistical values remain unchanged</u>.
- Probabilistic forecasts are available for longer lead times (15d), which allows an earlier detection of critical events.

- Chance constraints implemented as soft constraints within objective function. Hard chance constraints are currently under development.

Thank you...

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