Bayesian analysis of stage-discharge relationship affected by hysteresis and quantification of the associated uncertainties

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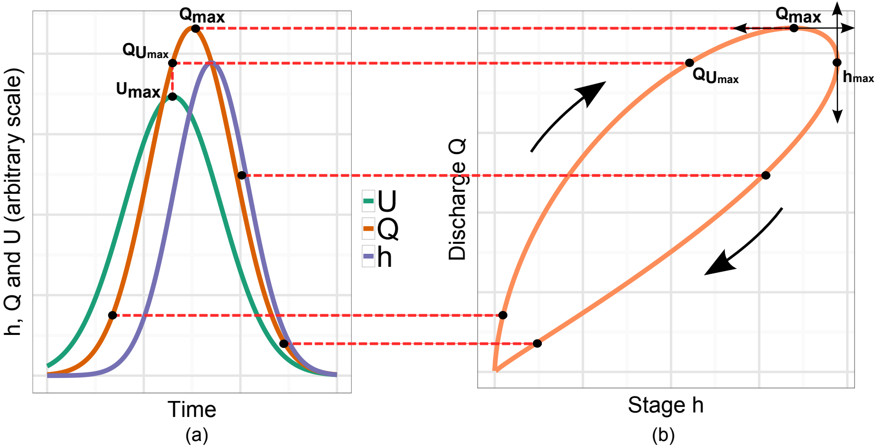
Definition



Qmax
QUmax
Umax
h, Q and U (arbitrary scale)
Time
(a)
U
Q
h
Qmax
QUmax
hmax
Discharge Q
Stage h
(b)

Rating curve models

- General formula :

$$Q\left(h, \frac{dh}{dt}, \frac{d^2h}{dt^2}, \frac{d^3h}{dt^3}\right) = K_S A (R_h)^{2/3} \sqrt{S_0 + \frac{1}{c}\frac{dh}{dt} - \frac{D}{c^3}\frac{d^2h}{dt^2} - \frac{G}{c^5}\frac{d^3h}{dt^3}}$$

- Manning-Strickler;
- Jones;
- Fenton (2nd order);
- Fenton (3rd order);

where

- h $[\mathrm{m}]$ is the stage;
- K_S $[\mathrm{m}^{1/3}.\mathrm{s}^{-1}]$ is the Strickler coefficient modelling the roughness of the riverbed;
- A $[\mathrm{m}^2]$ is the wetted surface;
- R_h $[\mathrm{m}]$ is the hydraulic radius;
- c $[\mathrm{m}.\mathrm{s}^{-1}]$ is the kinematic wave celerity;
- S_0 $[-]$ is the channel bed slope;
- D $[\mathrm{m}.\mathrm{s}^{-1}]$ is a coefficient (no additional parameters);
- G $[\mathrm{m}^2.\mathrm{s}^{-2}]$ is a coefficient (2 additional parameters);

Kinematic wave celerity c

- Assuming that the channel is uniform:

$$c = \frac{1}{B}\frac{\partial Q_r}{\partial h}$$

  where:
  - B $[\mathrm{m}]$ is the width of the channel;
  - Q_r $[\mathrm{m}^3.\mathrm{s}^{-1}]$ is the rated discharge;
- Thus, c can be modelled either as a function of stage or as a constant.

BaRatin (Bayesian Rating curve)

- **Required data:**
  - Gaugings with uncertainties;
  - Stage time series;
- **BaRatin recipe:**
  1) Rating curve formulation;
  2) Bayesian simulation;
  3) Discharge estimation;
- **Poster presentation:**

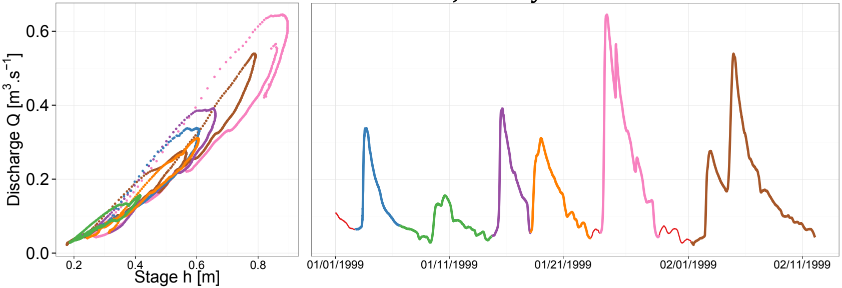
  Le Coz et *al.*: "Quantifying the uncertainty in discharge data using hydraulic knowledge and uncertain gaugings: a Bayesian method named BaRatin"

  $\Rightarrow$ Board R4, today between **17:30** and **19:00**.
- **Adaptation to hysteresis.**

The hydrometric station of A1

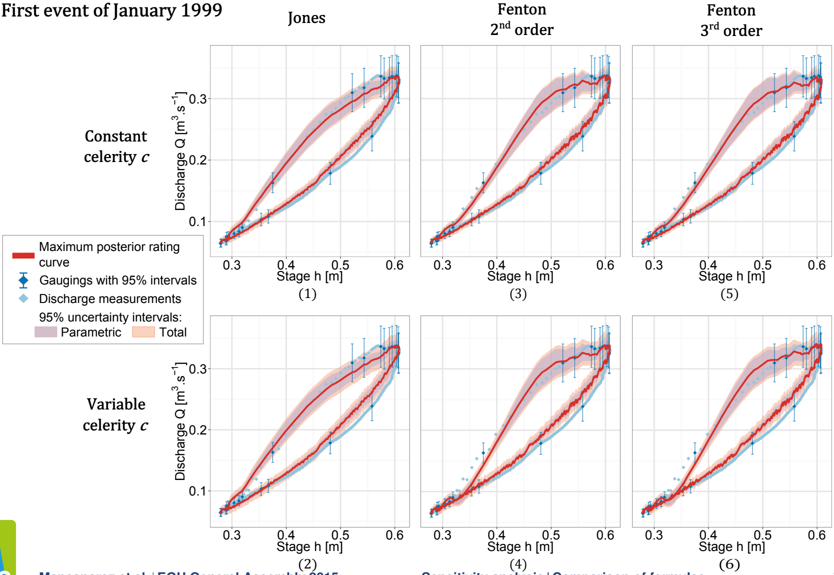
- Gauging flume, near Plymouth, North Carolina, USA;
- A continuous Doppler velocimeter in a calibrated cross-section;
- One hydrological year (1998-1999) of events with several hysteresis events:

First events of January 1999

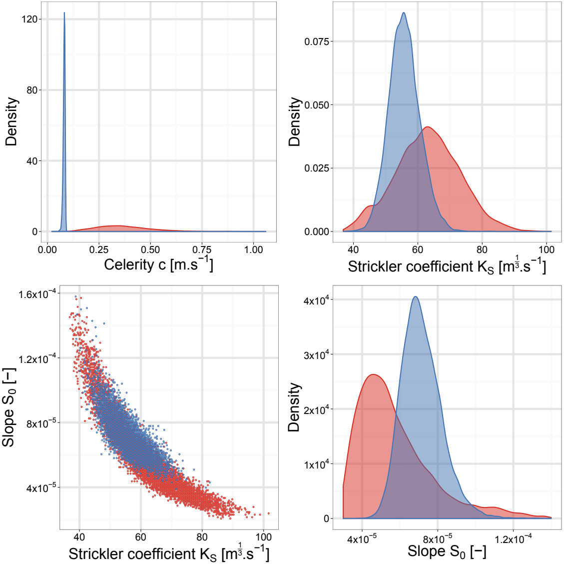


Comparison of formulas

First event of January 1999



Celerity *c* as a constant or not



First event of January 1999

Jones formula

- Constant celerity c
- Variable celerity c

Correlations

$cor(K_S,S_0) = -0.91$

$cor(K_S,S_0) = -0.79$

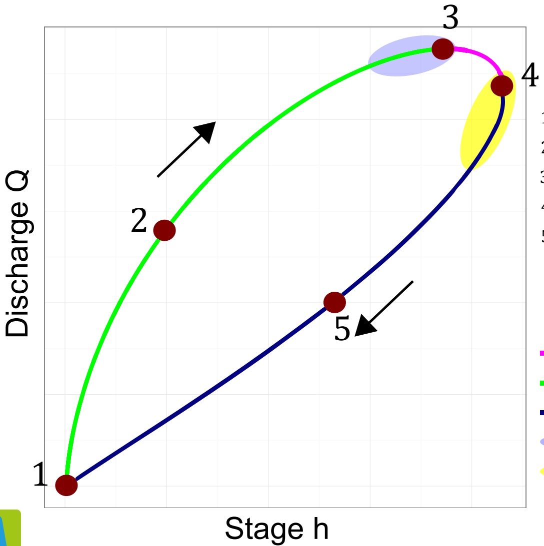
$cor(K_S,c) = 0.86$

$|cor(K_S,c)| < 0.6$

$cor(S_0,c) = -0.8$

$|cor(S_0,c)| < 0.3$

Gauging strategies



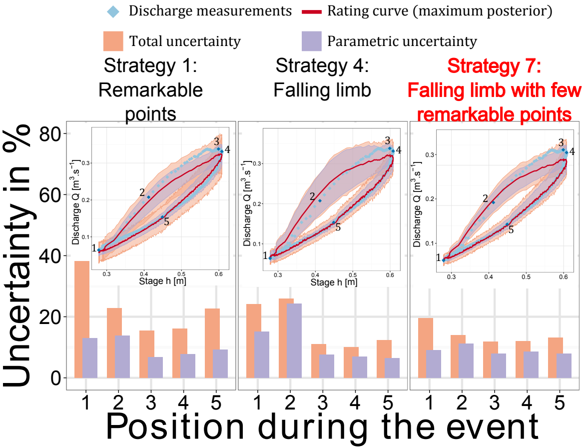
Remarkable points

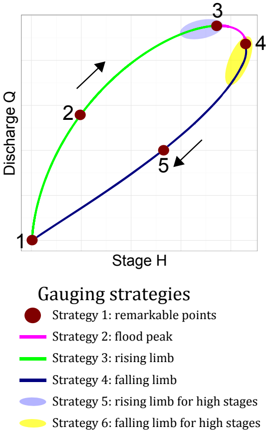
1: minimum of stage h
2: maximum of stage gradient dh/dt
3: maximum of discharge Q
4: maximum of stage h
5: minimum of stage gradient dh/dt

Gauging strategies

Strategy 1: remarkable points
Strategy 2: flood peak
Strategy 3: rising limb
Strategy 4: falling limb
Strategy 5: rising limb for high stages
Strategy 6: falling limb for high stages

Gauging strategies





Conclusions

- More precise hydraulic priors and/or less uncertain gaugings provide a better goodness-of-fit of the rating curve and smaller uncertainty;
- The simple Jones formula leads to as good results as the more complex Fenton formula;
- The variable celerity option brings less uncertain results than the constant celerity option;
- Calibration of hysteretic rating curve model can be made on different events;
- The best gauging strategy is to gauge near few remarkable points of the flood wave and use gaugings of the falling limb, not necessarily in a single event.

Questions ?

Thank you for your attention