



Boundary condition optimal control problem in lava flow modelling

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We study a problem of steady-state fluid flow with known thermal conditions (e.g., measured temperature and the heat flux at the surface of lava flow) at one segment of the model boundary and unknown conditions at its another segment. This problem belongs to a class of boundary condition optimal control problems and can be solved by data assimilation from one boundary to another using direct and adjoint models. We derive analytically the adjoint model and test the cost function and its gradient, which minimize the misfit between the known thermal condition and its model counterpart. Using optimization algorithms, we iterate between the direct and adjoint problems and determine the missing boundary condition as well as thermal and dynamic characteristics of the fluid flow. The efficiency of optimization algorithms – Polak-Ribiere conjugate gradient and the limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS) algorithms – have been tested with the aim to get a rapid convergence to the solution of this inverse ill-posed problem. Numerical results show that temperature and velocity can be determined with a high accuracy in the case of smooth input data. A noise imposed on the input data results in a less accurate solution, but still acceptable below some noise level.