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Construction of Galerkin's matrix for elementary potentials and an ellipsoidal solution domain based on series developments and general relations between Legendre's functions of the first and the second kind: Application in Earth's gravity field studies

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The role of boundary value problems in physical geodesy and in Earth's gravity field studies in particular is well-known. The paper focuses on Neumann's problem formulated for the exterior of an oblate ellipsoid of revolution as this is considered a basis for an iteration solution of the linear gravimetric boundary value problem in the determination of the disturbing potential. The approach follows the concept of variational methods and the notion of the weak solution. Hence Galerkin's approximations are applied, which means that the solution of the problem is approximated by linear combinations of basis functions with scalar coefficients. Our aim is to discuss the construction of Galerkin's matrix for basis functions generated by elementary potentials. Possibly, the computation of the entries of Galerkin's matrix is expected to be very simple for the elementary functions like these. Nevertheless, the opposite is true. Ellipsoidal harmonics are applied as a natural tool and elementary potentials are expressed by means of series of ellipsoidal harmonics. The problem, however, is the summation of the series that represent the entries of Galerkin's matrix. It is difficult to reduce the number of summation indices since in the ellipsoidal case there is no analogue to the addition theorem known for spherical harmonics. This makes the computation of Galerkin's matrix rather demanding. Therefore, the straightforward application of series of ellipsoidal harmonics is complemented by deeper relations contained in the theory of ordinary differential equations of second order and Legendre's functions. Subsequently, hypergeometric functions and series are used. Moreover, within some approximations the entries are split into parts. Some of the resulting series may be summed relatively easily, apart from technical tricks. For the remaining series the summation needs more complex tools. It was converted to elliptic integrals. The approach made it possible to deduce a closed (though approximate) form representation of the entries in Galerkin's matrix. Note that in essence the result rests on concepts and methods of mathematical analysis. In the paper it is confronted with a direct numerical approach concerning the implementation and use of Legendre's functions. The computation of the entries is considerably more demanding in this case. Nevertheless, conceptually it avoids approximations. The discussion illustrates some specific features associated with function bases generated by elementary potentials in case of problems formulated for the ellipsoidal solution domain.