

Spectral evolution of weakly nonlinear random waves: kinetic description vs direct numerical simulations

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We study numerically the long-term evolution of water wave spectra without wind forcing, using three different models, aiming at understanding the role of different sets of assumptions.

The first model is the classical Hasselmann kinetic equation (KE). We employ the WRT code kindly provided by G. van Vledder. Two other models are new. As the second model, we use the generalised kinetic equation (gKE), derived without the assumption of quasi-stationarity. Thus, unlike the KE, the gKE is valid in the cases when a wave spectrum is changing rapidly (e.g. at the initial stage of evolution of a narrow spectrum). However, the gKE employs the same statistical closure as the KE. The third model is based on the Zakharov integrodifferential equation for water waves and does not depend on any statistical assumptions. Since the Zakharov equation plays the role of the primitive equation of the theory of wave turbulence, we refer to this model as direct numerical simulation of spectral evolution (DNS-ZE).

For initial conditions, we choose two narrow-banded spectra with the same frequency distribution (a JONSWAP spectrum with high peakedness $\gamma = 6$) and different degrees of directionality. These spectra are from the set of observations collected in a directional wave tank by Onorato et al (2009). Spectrum A is very narrow in angle (corresponding to $N = 840$ in the \cos^N directional model). Spectrum B is initially wider in angle (corresponds to $N = 24$). Short-term evolution of both spectra ($O(10^2)$ wave periods) has been studied numerically by Xiao et al (2013) using two other approaches (broad-band modified nonlinear Schrödinger equation and direct numerical simulation based on the high-order spectral method). We use these results to verify the initial stage of our DNS-ZE simulations. However, the advantage of the DNS-ZE method is that it allows to study long-term spectral evolution (up to $O(10^4)$ periods), which was previously possible only with the KE.

In the short-term evolution, we find a good agreement between our DNS-ZE results and simulations by Xiao et al (2013), both for the evolution of frequency spectra and for the directional spreading. In the long term, all three approaches demonstrate very close evolution of integral characteristics of spectra, approaching for large time the theoretical asymptotes of the self-similar stage of evolution. However, the detailed comparison of the spectral evolution shows certain notable differences. Both kinetic equations give virtually identical evolution of spectrum B, but in the case of initially nearly one-dimensional spectrum A the KE overestimates the amplitude of the spectral peak. Meanwhile, the DNS-ZE results show considerably wider spectra with less pronounced peak. There is a striking difference for the rate of spectral broadening, which is much larger for the gKE and especially for the KE, than for the DNS-ZE. We show that the rates of change of the spectra obtained with the DNS-ZE are proportional to the fourth power of nonlinearity, corresponding to the dynamical timescale of evolution, rather than the statistical timescale of both kinetic equations.