



2D Potential theory using complex functions and conformal mapping

Pauline Le Maire (1,2) and Marc Munsch (1)

(1) Institut de Physique du Globe de Strasbourg, UMR 7516, Université de Strasbourg / EOST, CNRS, 1 rue Blessig, 67084 Strasbourg Cedex, FRANCE (pauline.le-maire@etu.unistra.fr), (2) Cardem, 7 Rue de l'Uranium 67800 Bischheim, France

For infinitely horizontally extended bodies, functions that describe potential and field equations (gravity and magnetics) outside bodies are 2D and harmonic. The consequence of this property is that potential and field equations can be written as complex analytic functions. We define these complex functions whose real part is the commonly used real function and imaginary part is its Hilbert transform. Using data or synthetic cases the transformation is easily performed in the Fourier domain by setting to zero all values for negative frequencies.

Written as complex functions of the complex variable, equations of potential and field in gravity and magnetics for different kinds of geometries are simple and correspond to powers of the inverse of the distance. For example, it is easily shown that for a tilted dyke, the dip and the apparent inclination have the same effect on the function and consequently that it is not possible, with data, to compute one of both values without knowing the other.

Conformal mapping is an original way to display potential field functions. Considering that the complex variable corresponds to the real axis, complex potential field functions resume to a limaçon, a curve formed by the path of the point fixed to a circle when that circle rolls around the outside of another circle. For example, the point corresponding to the maximum distance to the origin of the complex magnetic field due to a cylinder, corresponds to the maximum of the analytic signal as defined by Nabighan in 1972 and its phase corresponds to the apparent inclination.

Several applications are shown in different geological contexts using aeromagnetic data.