



## **The compressible adjoint equations in geodynamics: equations and numerical assessment**

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The adjoint method is a powerful means to obtain gradient information in a mantle convection model relative to past flow structure. While the adjoint equations in geodynamics have been derived for the conservation equations of mantle flow in their incompressible form, the applicability of this approximation to Earth is limited, because density increases by almost a factor of two from the surface to the Core Mantle Boundary. Here we introduce the compressible adjoint equations for the conservation equations in the anelastic-liquid approximation. Our derivation applies an operator formulation in Hilbert spaces, to connect to recent work in seismology (Fichtner et al (2006)) and geodynamics (Horbach et al (2014)), where the approach was used to derive the adjoint equations for the wave equation and incompressible mantle flow. We present numerical tests of the newly derived equations based on twin experiments, focusing on three simulations. A first, termed Compressible, assumes the compressible forward and adjoint equations, and represents the consistent means of including compressibility effects. A second, termed Mixed, applies the compressible forward equation, but ignores compressibility effects in the adjoint equations, where the incompressible equations are used instead. A third simulation, termed Incompressible, neglects compressibility effects entirely in the forward and adjoint equations relative to the reference twin. The compressible and mixed formulations successfully restore earlier mantle flow structure, while the incompressible formulation yields noticeable artifacts. Our results suggest the use of a compressible formulation, when applying the adjoint method to seismically derived mantle heterogeneity structure.