Nonlinear disintegration of sine wave in the framework of the Gardner equation

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Nonlinear disintegration of sine wave is studied in the framework of the Gardner equation (extended version of the Korteweg – de Vries equation with both quadratic and cubic nonlinear terms). Undular bores appear here as an intermediate stage of wave evolution. Our numerical computations demonstrate the features of undular bore developing for different signs of the cubic nonlinear term. If cubic nonlinear term is negative, and initial wave amplitude is large enough, two undular bores are generated from the two breaking points formed on both crest slopes (within dispersionless Gardner equation). Undular bore consists of one table-top soliton and a group of small soliton-like waves passing through the table-top soliton. If the cubic nonlinear term is positive and again the wave amplitude is large enough, the breaking points appear on crest and trough generating groups of positive and negative solitary-like pulses. It is shown that nonlinear interaction of waves happens according to one of scenarios of two-soliton interaction of “exchange” or “overtake” types with a phase shift. If small-amplitude pulses interact with large-amplitude soliton-like pulses, their speed in average is negative in the case when “free” velocity is positive. Nonlinear interaction leads to the generation of higher harmonics and spectrum width increases with amplitude increase independently of the sign of cubic nonlinear term. The breaking asymptotic k⁴/₃ predicted within the dispersionless Gardner equation emerges during the process of undular bore development. The formation of soliton-like perturbations leads to appearance of several spectral peaks which are downshifting with time.