

Applied Analytical Methods for Solving Some Problems of Wave Propagation in the Coastal Areas

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Analytical methods, easy for application, are proposed for the solution of the following four classical problems of coastline hydro mechanics:

1. Refraction of waves on coast slopes of arbitrary steepness;
2. Wave propagation in tapering water areas;
3. Longitudinal waves in open channels;
4. Long waves on uniform and non-uniform flows of water.

The first three of these problems are solved by the direct Galerkin-Kantorovich method with a choice of basic functions which completely satisfy all boundary conditions. This approach leads to obtaining new evolutionary equations which can be asymptotically solved by the WKB method.

The WKB solution of the first problem enables us to easily determine the three-dimensional field of velocities and to construct the refraction picture of the wave surface near the coast having an arbitrary angle of slope to the horizon varying from 0° to 180° . This solution, in particular for a vertical cliff, fully agrees with Stoker's particular but difficult solution. Moreover, it is shown for the first time that our Schrödinger type evolutionary equation leads to the formation of the so-called "potential wells" if the angle of coast slope to the horizon exceeds 45° , while the angle given at infinity (i.e. at a large distance from the shore) between the wave crests and the coastline exceeds 75° . This theoretical result expressed in terms of elementary functions is well consistent with the experimental observations and with lot of aerial photographs of waves in the coastal zones of the oceans [1,2]. For the second problem we introduce the notions of "wide" and "narrow" water areas. It is shown that Green's law on the wave height growth holds only for the narrow part of the water area, whereas in the wide part the tapering of the water area leads to an insignificant decrease of the wave height.

For the third problem, the bank slopes of trapezoidal channels are assumed to have an arbitrary angle of steepness. So far we have known the practically applicable solutions (obtained by MacDonald and Kelland) only for triangular channels whose lateral slopes to the horizon are 30° and 45° .

For the fourth problem, a number of unique results are obtained by the correct linearization of shallow water equations. These results include in particular the following: the wave propagation against the flow is blocked by a stream with a Froude number $Fr > 2/3$, but not with $Fr > 1$, as thought previously. New relations are derived for the conjugate depths of all types of hydraulic jumps and discontinuous roll-waves.

References:

1. Stoker, J.J. 1957 Water waves. The mathematical theory with application. New York: Interscience Publ., 567 p., (Figures 5.6.2, 5.6.3 and 5.6.5).
2. Hodgins, D.O., Le Blond, P.H. and Huntley, D.A., 1985, Shallow-water wave calculations. Canadian Contractor Report of Hydrography and Ocean Sciences, 10, 75 p., (Figure 3.5).

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