

Using the Bi-Orthogonal Decomposition framework to compute the three dimensional Empirical Orthogonal Functions of stratospheric planetary waves from time correlation matrices

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Many geophysical waves in the atmosphere or in the ocean have a three dimensional structure and contain a range of scales. This is for instance the case of planetary waves in the stratosphere connected to baroclinic eddies in the troposphere [1]. In the study of such waves from reanalysis data or output of numerical simulations, Empirical Orthogonal Functions (EOF) obtained as a Proper Orthogonal Decomposition of the data sets have been of great help. However, most of these computations rely on the diagonalisation of space correlation matrices: this means that the considered data set can only have a limited number of gridpoints. The main consequence is that such analyses are often only performed in planes (as function of height and latitude, or longitude and latitude for instance), which makes the educing of the three dimensional structure of the wave quite difficult. In the case of the afore mentioned waves, the matter of the longitudinal dependence or the proper correlation between modes through the tropopause is an open question.

An elegant manner to circumvent this problem is to consider the output of the Orthogonal Decomposition as a whole. Indeed, it has been shown that the normalised time series of the amplitude of each EOF, far from just being decorrelated from one another, are actually another set of orthogonal functions. These can actually be computed through the diagonalisation of the time correlation matrix of the data set, just like the EOF were the result of the diagonalisation of the space correlation matrix. The signal is then fully decomposed in the framework of the Bi-Orthogonal Decomposition as the sum of the n^{th} explained variance, time the n^{th} eigenmode of the time correlation times the n^{th} eigenmode of the spacial correlations [2,3]. A practical consequence of this result is that the EOF can be reconstructed from the projection of the dataset onto the eigenmodes of the time correlation matrix in the so-called snapshot method [4]. This is very practical when one wants to compute leading variance three dimensional EOFs which have tens of thousands (or more) grids points in a dataset which has a smaller length (up to a few thousands time frames for instance). Meanwhile traditional EOF are adapted to datasets which have smaller number of gridpoints but can have arbitrary length in time.

We propose a demonstration of this approach to compute the three dimensionnal leading EOF patterns corresponding to the planetary waves of our interest [1]. For this matter, we use the recently developped General Circulation Model ICON [5] to generate winter datasets with an orography triggering the desired waves. From the output, we compute the time correlation matrices and use them to reconstruct the leading EOFs of zonal velocity and temperature. From this, we can not only identify the pair of mode describing the propagation of the planetary waves, which gives us full information on their longitudinal dependence, as well as the vertical dependence both in the troposphere and the stratosphere. Among other things, the modes inform us precisely on the sharp temperature gradient of the wave at the tropopause. A similar investigation can also be performed on reanalysis datasets. It is hoped that obtaining this precise three dimensional structure will help the investigation of the mechanism of coupling between the tropospheric eddies and the stratospheric planetary waves.

References

- [1] C. J. Chan, R. A. Plumb, *The response to stratosphere forcing and its dependence on the state of the troposphere*, J. Atmo. Sci. **66**, 2107–2115 (2009).
- [2] N. Aubry, *The hidden beauty of proper orthogonal decomposition*, Theoret. Comput. Fluid Dyn. **2**, 339–352 (1991).
- [3] N. Aubry, R. Guyonnet, R. Lima, *Spatiotemporal analysis of complex signals: theory and applications*, J. Stat. Phys. **64** 683–739 (1991).

- [4] L. Sirovich, *Turbulence and the dynamics of coherent structures. I coherent structures*, Q. Appl. Math. **45**, 561–571 (1987).
- [5] G. Zängl, D. Reinert, P. Ripodas, M. Baldauf, *The ICON (ICOsahedral Non-hydrostatic) modelling framework of DWD and MPI-M: Description of the non-hydrostatic dynamical core*, Q. J. R. Meteorol. Soc. **141**, 563–579 (2015).