# Handling boundaries with the recursive filter 

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In variational data assimilation, a crucial task is to determine the background error covariance matrix $B$. The effect of $B$ on a field is often modelled through a series of operators among which a correlation operator. The recursive filter, when properly normalised to ensure the maximum of the solution is one, is a convenient correlation operator and is widely used as such. Often, multi-dimensional operators are constructed from the product of onedimensional operators. When its coefficients are calculated appropriately, the normalised one-dimensional first order recursive filter applied $N$ times models an autoregressive function of order $N$.

In ocean data assimilation an extra difficulty is to handle, within the correlation operator, the boundaries formed by the different coastlines or the bathymetry. Handling the East-West wrapping or polar folds in global configurations are also part of the issue. Generally, these problems are dealt with by calculating proper normalisation factors using costly methods, or extending the grid near boundaries. We will show that the normalisation factor can be calculated inexpensively through an analytical formula with a corrective term to handle properly the boundary when Neumann or Dirichlet boundary conditions are used.
In its classical formulation however, the recursive filter uses Robin (third type) boundary conditions. We will show that this formulation can be slightly modified in order to account for Neumann, Dirichlet or periodic boundary conditions. To do so, extra coefficients must be calculated through a simple recursive formula. In a global $1 / 4$ degree framework, this new formulation is shown to be less expensive and more accurate than the classical formulation associated with an appropriate grid extension.

