



An efficient distribution method for nonlinear transport problems in highly heterogeneous stochastic porous media

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Because geophysical data are inexorably sparse and incomplete, stochastic treatments of simulated responses are crucial to explore possible scenarios and assess risks in subsurface problems. In particular, nonlinear two-phase flows in porous media are essential, yet challenging, in reservoir simulation and hydrology. Adding highly heterogeneous and uncertain input, such as the permeability and porosity fields, transforms the estimation of the flow response into a tough stochastic problem for which computationally expensive Monte Carlo (MC) simulations remain the preferred option. We propose an alternative approach to evaluate the probability distribution of the (water) saturation for the stochastic Buckley-Leverett problem when the probability distributions of the permeability and porosity fields are available. We give a computationally efficient and numerically accurate method to estimate the one-point probability density (PDF) and cumulative distribution functions (CDF) of the (water) saturation. The distribution method draws inspiration from a Lagrangian approach of the stochastic transport problem and expresses the saturation PDF and CDF essentially in terms of a deterministic mapping and the distribution and statistics of scalar random fields. In a large class of applications these random fields can be estimated at low computational costs (few MC runs), thus making the distribution method attractive. Even though the method relies on a key assumption of fixed streamlines, we show that it performs well for high input variances, which is the case of interest. Once the saturation distribution is determined, any one-point statistics thereof can be obtained, especially the saturation average and standard deviation. Moreover, the probability of rare events and saturation quantiles (e.g. P10, P50 and P90) can be efficiently derived from the distribution method. These statistics can then be used for risk assessment, as well as data assimilation and uncertainty reduction in the prior knowledge of input distributions. We provide various examples and comparisons with MC simulations to illustrate the performance of the method.