

Mean flow generation mechanism by inertial waves and normal modes

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The mean flow generation mechanism by nonlinearity of the inertial normal modes and inertial wave beams in a rotating annular cavity with longitudinally librating walls in stable regime is discussed.

Inertial normal modes (standing waves) are excited when libration frequency matches eigenfrequencies of the system. Inertial wave beams are produced by Ekman pumping and suction in a rotating cylinder and form periodic orbits or periodic ray trajectories at selected frequencies. Inertial wave beams emerge as concentrated shear layers in a librating annular cavity, while normal modes appear as global recirculation cells. Both (inertial wave beam and mode) are helical and thus intrinsically non-linear flow structures. No second mode or wave is necessary for non-linearity. We considered the low order normal modes (1,1), (2,1) and (2,2) which are expected to be excited in the planetary objects and investigate the mean flow generation mechanism using two independent solutions: 1) analytical solution (Borcia 2012) and 2) the wave component of the flow (ω_0 component) obtained from the direct numerical simulation (DNS). It is well known that a retrograde bulk mean flow is generated by the Ekman boundary layer and $E^{1/4}$ -Stewartson layer close to the outer cylinder side wall due to libration. At and around the normal mode resonant frequencies we found additionally a prograde azimuthal mean flow (Inertial Normal Mode Mean Flow: INMMF) in the bulk of the fluid. The fluid in the bulk is in geostrophic balance in the absence of the inertial normal modes. However, when INMMF is excited, we found that the geostrophic balance does not hold in the region occupied by INMMF. We hypothesize that INMMF is generated by the nonlinearity of the normal modes or by second order effects. Expanding the velocity $V(u_r, u_\theta, u_z)$ and pressure (p) in a power series in ϵ (libration amplitude), the Navier-Stokes equations are segregated into the linear and nonlinear parts at orders ϵ^1 and ϵ^2 , respectively. The former is used to find the analytical solution of the normal modes (Borcia 2012). Plugging two independent solutions into the latter we investigate the generation mechanism of INMMF. We found $R1^1 = \partial_z(u_r^1 u_z^1)$, $R2^1 = \partial_r(u_r^1 u_r^1)$ as source terms responsible for the generation of INMMF. The helical structure of the inertial waves causes the nonlinear terms $R1$ and $R2$ to be nonzero, contributing to the generation of INMMF. We used u_r^a and u_z^a obtained from the analytical solution (Borcia 2012) and computed the source terms $R1^a$ and $R2^a$ and found a structural correspondence with the corresponding field computed from the DNS solution for the three normal modes investigated. The sum of $R1^1$ and $R2^1$ exhibits a good structural correspondence with INMMF. Interestingly, INMMF magnitude depends on the inertial wave beams and normal modes. For instance we found that INMMF is generated more efficiently for the libration frequency $\omega = 1.58$, although the resonant frequency is predicted by the analytical solution to be at $\omega = 1.576$ (normal mode (2,1)). Separating the inertial wave beams from the flow field obtained by DNS, using the analytical normal mode solution, we explored the phase lag between inertial wave beams and normal mode. We inferred that the normal mode amplitude is high only if the phase lag between the inertial wave beam and the normal mode is predominantly positive. In this case a high amplitude INMMF amplitude can be found. This supports the hypothesis that the normal modes are generated by the inertial wave beam in analogy to resonant forcing in classical mechanics. Interestingly, the ‘optimum’ phase lag found is much smaller than $\pi/2$.

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