

Two-way WKB Approximation Applied to GPR - COST Action TU1208

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The main goal of subsurface radio wave probing consists in reconstruction of the shape and the electrical properties of buried objects in material media. For this purpose the knowledge of the laws of EM pulse excitation and propagation in non-uniform subsurface medium is required, as well as the methods and algorithms of solving the inverse problem. Two ways of treating this problem exist. On the one hand, one can describe EM wave propagation by solving the Maxwell's equations with finite difference methods implemented in computer codes. However, when solving inverse problems, pure numerical algorithms require huge amount of calculation and, as a consequence, long calculation time. In this respect, more promising are analytical approaches. Here, we apply couple wave theory ("two-way WKB" approximation) to the problem of subsurface wave propagation. The derived formulas can be used in GPR design and for fast data processing of the experimental data.

We start from the 1D model problem of GPR probing. Classical WKB method [1] allows one to describe wave propagation through non-uniform media with slowly varying dielectric permittivity. A principal shortcoming of this approximation is that it does not take into account backward reflection from permittivity gradients. Consequently, WKB method as such can not be used for the purposes of GPR sounding. An extension of this approximation consists in solving two coupled WKB-type equations by iterations. This approach properly describes backward reflections and provides good accuracy in a wide frequency range [2]. In our previous work [3] a time-domain counterpart of the Bremmer-Brekhovkikh approximation has been derived and applied to a 1D inverse problem of subsurface medium probing by an ultra-wide band EM pulse.

In order to convert this approach into a practical GPR algorithm, a more realistic model is required: 2D or 3D propagation from a localized source with the effects of wave divergence and refraction taken into account. In this work we study bistatic EM pulse probing of a horizontally layered medium in a 2D case. Coupled WKB equations set describing both forward and backward waves are derived and solved analytically.

The comparison of our semi-analytical solutions with numerical calculations by gprMax software [4] demonstrates a good agreement, being hundreds of times faster than the latter. Our numerical results explain the protracted return pulses in the low-frequency GPR data. As an example, we discuss the experimental data obtained during the GPR mission in search of a big fragment of Chelyabinsk meteorite under a thick silt layer at the bottom of Chebarcul' Lake [5].

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