



Higher-order nonlinear Schrodinger equations for simulations of surface wavetrains

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Numerous recent results of numerical and laboratory simulations of waves on the water surface claim that solutions of the weakly nonlinear theory for weakly modulated waves in many cases allow a smooth generalization to the conditions of strong nonlinearity and dispersion, even when the ‘envelope’ is difficult to determine. The conditionally ‘strongly nonlinear’ high-order asymptotic equations still imply the smallness of the parameter employed in the asymptotic series. Thus at some (unknown a priori) level of nonlinearity and / or dispersion the asymptotic theory breaks down; then the higher-order corrections become useless and may even make the description worse.

In this paper we use the higher-order nonlinear Schrodinger (NLS) equation, derived in [1] (the fifth-order NLS equation, or next-order beyond the classic Dysthe equation [2]), for simulations of modulated deep-water wave trains, which attain very large steepness (below or beyond the breaking limit) due to the Benjamin – Feir instability. The results are compared with fully nonlinear simulations of the potential Euler equations as well as with the weakly nonlinear theories represented by the nonlinear Schrodinger equation and the classic Dysthe equation with full linear dispersion [2]. We show that the next-order Dysthe equation can significantly improve the description of strongly nonlinear wave dynamics compared with the lower-order asymptotic models.

[1] A.V. Slunyaev, A high-order nonlinear envelope equation for gravity waves in finite-depth water. JETP 101, 926–941 (2005).

[2] K. Trulsen, K.B. Dysthe, A modified nonlinear Schrödinger equation for broader bandwidth gravity waves on deep water. Wave Motion 24, 281–289 (1996).