



The Inverse Problem of Klein-Gordon Equation Boundary Value Problem and Its Application in Data Assimilation

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It is often difficult to provide the exact boundary condition in the practical use of variational method. The Euler equation derived from variational method cannot be solved without boundary condition. However, in some application problems such as the assimilation of remote sensing data, the values can be easily obtained in the inner region of the domain. Since the solution of elliptic partial differential equations continuously depends on the boundary condition, the boundary condition can be retrieved using part solutions in the inner area. In this paper, the variational problem of remote sensing data assimilation within a circular area is first established. The Klein-Gordon elliptic equation is derived from the Euler method of variational problems with assumed boundary condition. Secondly, a computer-friendly Green function is constructed for the Dirichlet problem of two-dimensional Klein-Gordon equation, with the formal solution according to Green formula. Thirdly, boundary values are retrieved by solving the optimal problem which is constructed according to the smoothness of boundary value function and the best approximation between formal solutions and high-accuracy measurements in the interior of the domain. Finally, the assimilation problem is solved on substituting the retrieved boundary values into the Klein-Gordon equation. It is a type of inverse problem in mathematics. The advantage of our method lies in that it needs no assumptions of the boundary condition. It thus alleviates the error introduced by artificial boundary condition in data fusion using variational method in the past.