

## Distribution of kriging errors, the implications and how to communicate them

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Kriging in one form or another has become perhaps the most popular method for spatial prediction in environmental science. Each prediction is unbiased and of minimum variance, which itself is estimated. The kriging variances depend on the mathematical model chosen to describe the spatial variation; different models, however plausible, give rise to different minimized variances. Practitioners often compare models by so-called cross-validation before finally choosing the most appropriate for their kriging. One proceeds as follows. One removes a unit (a sampling point) from the whole set, krige the value there and compares the kriged value with the value observed to obtain the deviation or error. One repeats the process for each and every point in turn and for all plausible models. One then computes the mean errors (MEs) and the mean of the squared errors (MSEs). Ideally a squared error should equal the corresponding kriging variance ( $\sigma_K^2$ ), and so one is advised to choose the model for which on average the squared errors most nearly equal the kriging variances, i.e. the ratio  $MSDR = MSE/\sigma_K^2 \approx 1$ .

Maximum likelihood estimation of models almost guarantees that the MSDR equals 1, and so the kriging variances are unbiased predictors of the squared error across the region. The method is based on the assumption that the errors have a normal distribution. The squared deviation ratio (SDR) should therefore be distributed as  $\chi^2$  with one degree of freedom with a median of 0.455. We have found that often the median of the SDR (MedSDR) is less, in some instances much less, than 0.455 even though the mean of the SDR is close to 1. It seems that in these cases the distributions of the errors are leptokurtic, i.e. they have an excess of predictions close to the true values, excesses near the extremes and a dearth of predictions in between. In these cases the kriging variances are poor measures of the uncertainty at individual sites. The uncertainty is typically under-estimated for the extreme observations and compensated for by over estimating for other observations. Statisticians must tell users when they present maps of predictions.

We illustrate the situation with results from mapping salinity in land reclaimed from the Yangtze delta in the Gulf of Hangzhou, China. There the apparent electrical conductivity ( $EC_a$ ) of the topsoil was measured at 525 points in a field of 2.3 ha. The marginal distribution of the observations was strongly positively skewed, and so the observed  $EC_a$ s were transformed to their logarithms to give an approximately symmetric distribution. That distribution was strongly platykurtic with short tails and no evident outliers. The logarithms were analysed as a mixed model of quadratic drift plus correlated random residuals with a spherical variogram. The kriged predictions that deviated from their true values with an MSDR of 0.993, but with a medSDR=0.324. The coefficient of kurtosis of the deviations was 1.45, i.e. substantially larger than 0 for a normal distribution.

The reasons for this behaviour are being sought. The most likely explanation is that there are spatial outliers, i.e. points at which the observed values that differ markedly from those at their their closest neighbours.