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Spatial and temporal compact equations for water waves

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Spatial and temporal compact equations for water waves

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A one-dimensional potential flow of an ideal incompressible fluid with a free surface in a gravity field is the Hamiltonian system with the Hamiltonian:

$$H = \frac{1}{2}\int dx \int_{-\infty}^{\eta} |\nabla\phi|^2 dz + \frac{g}{2}\int \eta^2 dx$$

$\phi(x,z,t)$ - is the potential of the fluid, g - gravity acceleration, $\eta(x,t)$ - surface profile

Hamiltonian can be expanded as infinite series of steepness:

$$\begin{aligned} H &= H_2 + H_3 + H_4 + \dots \\ H_2 &= \frac{1}{2}\int (g\eta^2 + \psi\hat{k}\psi)dx, \\ H_3 &= -\frac{1}{2}\int \{(\hat{k}\psi)^2 - (\psi_x)^2\}\eta dx, \\ H_4 &= \frac{1}{2}\int \{\psi_{xx}\eta^2\hat{k}\psi + \psi\hat{k}(\eta\hat{k}(\eta\hat{k}\psi))\}dx. \end{aligned} \tag{1}$$

where $\hat{k}$ corresponds to the multiplication by $|k|$ in Fourier space, $\psi(x,t) = \phi(x,\eta(x,t),t)$. This truncated Hamiltonian is enough for gravity waves of moderate amplitudes and can not be reduced.

We have derived self-consistent compact equations, both spatial and temporal, for unidirectional water waves.

Equations are written for normal complex variable $c(x,t)$, not for $\psi(x,t)$ and $\eta(x,t)$.

Hamiltonian for temporal compact equation can be written in x-space as following:

$$H = \int c^*\hat{V}c\,dx + \frac{1}{2}\int \left[\frac{i}{4}(c^2\frac{\partial}{\partial x}c^{*2} - c^{*2}\frac{\partial}{\partial x}c^2) - |c|^2\hat{K}(|c|^2)\right]dx \tag{2}$$

Here operator $\hat{V}$ in K-space is so that $V_k = \frac{\omega_k}{k}$. If along with this to introduce Gardner-Zakharov-Faddeev bracket (for the analytic in the upper half-plane function)

$$\partial_x^+ \Leftrightarrow ik\theta_k \tag{3}$$

Hamiltonian for spatial compact equation is the following:

$$\begin{aligned} & H = \frac{1}{g}\int \frac{1}{\omega}|c_\omega|^2 d\omega + \\ + \quad & \frac{1}{2g^3}\int |c|^2(\ddot{c}^* c + \ddot{c}c^*)dt + \frac{i}{g^2}\int |c|^2\hat{\omega}(\dot{c}c^* - c\dot{c}^*)dt. \end{aligned} \tag{4}$$

equation of motion is:

$$\begin{aligned} & \frac{\partial}{\partial x}c + \frac{i}{g}\frac{\partial^2}{\partial t^2}c = \\ = \quad & \frac{1}{2g^3}\frac{\partial^3}{\partial t^3}\left[\frac{\partial^2}{\partial t^2}\left(|c|^2 c\right) + 2|c|^2\ddot{c} + \ddot{c}^* c^2\right] + \\ + \quad & \frac{i}{g^3}\frac{\partial^3}{\partial t^3}\left[\frac{\partial}{\partial t}\left(c\hat{\omega}|c|^2\right) + \dot{c}\hat{\omega}|c|^2 + c\hat{\omega}\left(\dot{c}c^* - c\dot{c}^*\right)\right]. \end{aligned} \tag{5}$$

It solves the spatial Cauchy problem for surface gravity wave on the deep water.

Main features of the equations are:

- Equations are written for complex normal variable $c(x,t)$ which is analytic function in the upper half-plane
- Hamiltonians both for temporal and spatial equations are very simple
- It can be easily implemented for numerical simulation

The equations can be generalized for "almost" 2-D waves like KdV is generalized to KP.

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