

## **Hierarchical Bayes Ensemble Kalman Filter for geophysical data assimilation**

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In the Ensemble Kalman Filter (EnKF), the forecast error covariance matrix  $B$  is estimated from a sample (ensemble), which inevitably implies a degree of uncertainty. This uncertainty is especially large in high dimensions, where the affordable ensemble size is orders of magnitude less than the dimensionality of the system. Common remedies include ad-hoc devices like variance inflation and covariance localization. The goal of this study is to optimize the account for the inherent uncertainty of the  $B$  matrix in EnKF.

Following the idea by Myrseth and Omre (2010), we explicitly admit that the  $B$  matrix is unknown and random and estimate it along with the state ( $x$ ) in an optimal hierarchical Bayes analysis scheme. We separate forecast errors into predictability errors (i.e. forecast errors due to uncertainties in the initial data) and model errors (forecast errors due to imperfections in the forecast model) and include the two respective components  $P$  and  $Q$  of the  $B$  matrix into the extended control vector  $(x, P, Q)$ . Similarly, we break the traditional forecast ensemble into the predictability-error related ensemble and model-error related ensemble. The reason for the separation of model errors from predictability errors is the fundamental difference between the two sources of error. Model error are external (i.e. do not depend on the filter's performance) whereas predictability errors are internal to a filter (i.e. are determined by the filter's behavior).

At the analysis step, we specify Inverse Wishart based priors for the random matrices  $P$  and  $Q$  and conditionally Gaussian prior for the state  $x$ . Then, we update the prior distribution of  $(x, P, Q)$  using both observation and ensemble data, so that ensemble members are used as generalized observations and ordinary observations are allowed to influence the covariances. We show that for linear dynamics and linear observation operators, conditional Gaussianity of the state is preserved in the course of filtering.

At the forecast step, the new filter named Hierarchical Bayes Ensemble Kalman Filter (HBEF) employs a full-fledged secondary filter that cycles and updates the covariance matrices  $P$  and  $Q$ .

Approximations that lead to practicable analysis algorithms are proposed and tested. The HBEF is studied in numerical experiments with a one-variable model of "truth" and found significantly superior to EnKF in a wide range of filtering regimes. Results of numerical experiments with multi-variable models of truth are also presented.