



A Dynamic Eddy Viscosity Model for the Shallow Water Equations Solved by Spectral Element and Discontinuous Galerkin Methods

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We present the solution of the viscous shallow water equations where viscosity is built as a residual-based subgrid scale model originally designed for large eddy simulation of compressible [1] and stratified flows [2]. The necessity of viscosity for a shallow water model not only finds motivation from mathematical analysis [3], but is supported by physical reasoning as can be seen by an analysis of the energetics of the solution. We simulated the flow of an idealized wave as it hits a set of obstacles. The kinetic energy spectrum of this flow shows that, although the inviscid Galerkin solutions –by spectral elements and discontinuous Galerkin [4]– preserve numerical stability in spite of the spurious oscillations in the proximity of the wave fronts, the slope of the energy cascade deviates from the theoretically expected values. We show that only a sufficiently small amount of dynamically adaptive viscosity removes the unwanted high-frequency modes while preserving the overall sharpness of the solution. In addition, it yields a physically plausible energy decay.

This work is motivated by a larger interest in the application of a shallow water model to the solution of tsunami triggered coastal flows. In particular, coastal flows in regions around the world where coastal parks made of mitigation hills of different sizes and configurations are considered as a means to deviate the power of the incoming wave.

References

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