



New solutions for the confined horizontal aquifer

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The Boussinesq equation is a dynamical equation for the free surface of saturated subsurface flows over an impervious bed. Boussinesq equation is non-linear. The non-linearity comes from the reduction of the dimensionality of the problem: The flow is assumed to be vertically homogeneous, therefore the flow rate through a cross section of the flow is proportional to the free surface height times the hydraulic gradient, which is assumed to be equal to the slope of the free surface. In the present work we consider the case of the subsurface flow with horizontal bed. This is a case with an infinite Henderson and Wooding parameter, that is, it is the limiting case where the non-linear term is present in the Boussinesq equation while the linear spatial derivative term vanishes. Nonetheless, no analogue of the kinematic wave exists in this case as there is no exact solution for the build-up phase. Neither is there an exact recession-phase solution that holds in early times, as the Boussinesq separable solution is actually an asymptotic solution for large times. We construct approximate solutions for the horizontal aquifer which utilize directly the dynamical content of the non-linear Boussinesq equation. The approximate character of the solution lies in the fact that we start with a pre-supposed form for the solution, an educated guess, based on the nature of the initial condition as well as empirical observations from the numerical solution of the problem. The forms we shall use are power series of the location variable x along the bed with time-dependent coefficients. The series are not necessarily analytic. The boundary conditions are incorporated in the structure of the series from the beginning. The time-dependent coefficients are then determined by applying the Boussinesq equation and its spatial derivatives at the end-points of the aquifer. The forms are chosen also on the basis of their solubility; we would like to be able to construct explicitly the approximate solution. This way one derives explicit and fairly simple formulas which are excellent approximations of the solutions of the Boussinesq equation that one may obtain only numerically. These ideas essentially outline a general method, or philosophy, for the construction of special approximate solutions of the Boussinesq equation. We work out two specific examples. We construct approximate solutions of a single build-up and recession phase, starting from completely empty aquifer in the build-up phase and a steady state in the recession phase. We compare those solutions to numerical solutions of the Boussinesq equation and also with the Boussinesq separable exact solution for the recession phase and we show a perfect agreement.