

Sampling theorems and harmonic analysis of the gravitational field using gradiometer data

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It is well known that sampling theorems enable a precise reconstruction of band-limited continuous functions given on a grid.

Here we consider the possibility of using classical and relatively new sampling theorems on the sphere for implementation of harmonic analysis of the gravitational field of the Earth based on gradiometer data.

Let f be a function band-limited at degree N .

The Gauss-Legendre theorem allows to reconstruct f precisely by its sampled values given on $\sim 2N^2$ -grid (not equiangular by latitude).

The Driscoll-Healy theorem (Driscoll and Healy, 1994) requires $\sim 4N^2$ samples but on an equiangular grid to obtain the same result.

A novel sampling theorem by J. D. McEwen and Y. Wiaux (McEwen and Wiaux, 2011) states that the function f can be restored from its values on an equiangular $\sim 2N^2$ -grid. It requires $3(N - 1)$ less samples than the Gauss-Legendre approach.

The sampling scheme developed in (Khalid et al., 2014) on the sphere permits accurate computation of the spherical harmonic transform of f using only N^2 samples. It is not a sampling theorem in the strict sense since from the theory point of view it does not recover the function exactly, but it lets to achieve high accuracy even for very broad band-limits.

We performed numerical experiments to explore the accuracy of recovering unlimited functions on the sphere by means of sampling theorems.

For this purpose second derivatives of the gravity potential were computed using EGM2008 up to d/o 2160 on four different grids corresponding to the above mentioned sampling theorems with $N = 900$.

Harmonic analysis of these sampled derivatives gives their spherical harmonic coefficients.

We transformed them to potential coefficients using the formulae from (Petrovskaya and Vershkov, 2012) and compared the result with EGM2008.

As all the sampling schemes demonstrated almost the same results and permitted to recover an unlimited function up to d/o ~ 300 with very high accuracy, we are ready to recommend practical use of the optimal sampling scheme because of the smallest number of samples required.