

Direct observation of nonlinear coupling in wave turbulence at the surface of water and relevance of approximate resonances

Quentin Aubourg (1) and Nicolas Mordant (1,2)

(1) Université Grenoble Alpes, LEGI, CNRS F-38000 Grenoble, France (nicolas.mordant@ujf-grenoble.fr), (2) Institut Universitaire de France, 103, bd Saint Michel, F-75005 Paris, France

The theoretical framework of Weak Turbulence describes the statistical properties of a large collection of nonlinear waves. For a weakly nonlinear wave field, energy is assumed to be transferred only through resonant interaction. This enables the computation of analytical solutions of the stationary statistical states (Zakharov spectrum). Some similarities with hydrodynamical turbulence appear: an energy cascade is present from the injection scale to the dissipation at small scales. The theory has been applied to numerous systems many of them being of geophysical or astrophysical nature (water surface waves, internal waves, inertial waves, solar winds) as well as superfluid turbulence, lasers, nonlinear optics in fibers or vibrated elastic plates.

For water surface waves, experimental laboratory measurements often fail to reproduce quantitatively theoretical predictions. Gravity waves and capillary waves are often treated separately because of their different nature. For capillary waves, energy is supposed to be transferred through 3-waves interactions, whereas for gravity waves the coupling involves 4 waves (because of the curvature of the dispersion relation which does not allow triadic solutions). In the laboratory, the range of excited wavelengths are usually not strongly separated from the crossover between capillary and gravity waves (which occur near 13 Hz) due to size or measurement limitations. Near this crossover, the dispersion relation is significantly affected and this impacts most likely the theoretical predictions. To investigate how this special point may act on the phenomenology, we report laboratory experiments on gravity-capillary waves focused on the crossover (Aubourg, Mordant-PRL, 2015).

The setup consists in a $70 \times 40 \text{ cm}^2$ vessel where waves are generated by horizontal vibration. A Fourier Transform Profilometry technique is used that is fully resolved in time and space and thus permits to compute the full space-time spectrum. The presence of an energy cascade is clearly observed consistently with previous measurements. A large amount of data permits us to use higher order statistical tools to investigate directly the resonant interactions. We observe a strong presence of triadic interactions in our system, confirming the foundations of the weak wave turbulence theory. A significant part of these interactions are non-local and enable coupling between capillary and gravity waves. We also emphasize the role of approximate resonances that are made possible by the nonlinear spectral widening. The quasi-resonances increase significantly the number of wave interactions and in particular open the possibility of observing 3-wave coupling among gravity waves although 3-wave exact resonances are prohibited. These effects are being currently investigated in a larger size experiment using a 13m in diameter wave flume.

Our observation raises the question of the importance of these approximate resonances of gravity waves in energy transfers both in the theory and in the ocean.