



## **Topology of sustainable management of dynamical systems with desirable states: from defining planetary boundaries to safe operating spaces in the Earth System**

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To keep the Earth System in a desirable region of its state space, such as defined by the recently suggested “tolerable environment and development window”, “guardrails”, “planetary boundaries”, or “safe (and just) operating space for humanity”, one not only needs to understand the quantitative internal dynamics of the system and the available options for influencing it (management), but also the structure of the system’s state space with regard to certain qualitative differences. Important questions are: Which state space regions can be reached from which others with or without leaving the desirable region? Which regions are in a variety of senses “safe” to stay in when management options might break away, and which qualitative decision problems may occur as a consequence of this topological structure?

In this work, we develop a mathematical theory of the qualitative topology of the state space of a dynamical system with management options and desirable states, as a complement to the existing literature on optimal control which is more focussed on quantitative optimization and is much applied in both the engineering and the integrated assessment literature. We suggest a certain terminology for the various resulting regions of the state space and perform a detailed formal classification of the possible states with respect to the possibility of avoiding or leaving the undesired region. Our results indicate that before performing some form of quantitative optimization such as of indicators of human well-being for achieving certain sustainable development goals, a sustainable and resilient management of the Earth System may require decisions of a more discrete type that come in the form of several dilemmas, e.g., choosing between eventual safety and uninterrupted desirability, or between uninterrupted safety and larger flexibility.

We illustrate the concepts and dilemmas drawing on conceptual models from climate science, ecology, co-evolutionary Earth System modeling, economics, and classical mechanics, and discuss their potential relevance for the climate and sustainability debate, in particular suggesting several levels of planetary boundaries of qualitatively increasing safety.

### Reference:

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