



On the convergence of (ensemble) Kalman filters and smoothers onto the unstable subspace

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The characteristics of the model dynamics are critical in the performance of (ensemble) Kalman filters and smoothers. In particular, as emphasised in the seminal work of Anna Trevisan and co-authors, the error covariance matrix is asymptotically supported by the unstable and neutral subspace only, i.e. it is span by the backward Lyapunov vectors with non-negative exponents. This behaviour is at the heart of algorithms known as Assimilation in the Unstable Subspace, although its formal proof was still missing.

This convergence property, its analytic proof, meaning and implications for the design of efficient reduced-order data assimilation algorithms are the topics of this talk. The structure of the talk is as follows.

Firstly, we provide the analytic proof of the convergence on the unstable and neutral subspace in the linear dynamics and linear observation operator case, along with rigorous results giving the rate of such convergence. The derivation is based on an expression that relates explicitly the covariance matrix at an arbitrary time with the initial error covariance. Numerical results are also shown to illustrate and support the mathematical claims.

Secondly, we discuss how this neat picture is modified when the dynamics become nonlinear and chaotic and when it is not possible to derive analytic formulas. In this case an ensemble Kalman filter (EnKF) is used and the connection between the convergence properties on the unstable-neutral subspace and the EnKF covariance inflation is discussed. We also explain why, in the perfect model setting, the iterative ensemble Kalman smoother (IEnKS), as an efficient filtering and smoothing technique, has an error covariance matrix whose projection is more focused on the unstable-neutral subspace than that of the EnKF.

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