

## Stochastic bedload transport model: the remains of the day

Christophe Ancey (1) and Patricio Bohorquez (2)

(1) Ecole Polytechnique Fédérale de Lausanne, ENAC/ICARE/LHE, LAUSANNE, Switzerland (christophe.ancey@epfl.ch),

(2) Área de Mecánica de Fluidos, Universidad de Jaén, Campus de las Lagunillas, Jaén, Spain

Hans Einstein is associated with the earliest investigations into stochastic behaviour of bedload transport in the 1930s. While the stochastic approach has attracted growing attention over the years, the rate of progress has been slow. Recent advances have been made in theory of bedload transport and high-resolution measurements of sediment transport rates. This talk outlines the main achievements in our approach to bedload transport in mountain streams under low to moderate flow conditions.

The building block of our approach is an Eulerian description of the number  $N$  of moving particles in a fixed volume. An evolution equation for this number  $N$  can be derived using the framework of birth death Markov process. By generalizing this equation to an array of adjacent volumes, we end up with a stochastic formulation of the mass balance equation for the bedload. There is naturally a strong coupling between transported sediment, bed morphology, and flow conditions, which can be described using the shallow water equations supplemented by the stochastic mass balance equation (that can be considered a stochastic Exner equation). We therefore refer to the resulting system of equations as the Saint-Venant-Exner equations (SVEE).

We present some laboratory applications that are representative of mountain gravel-bed rivers. In particular, we focus on anti-dunes formation on steep slope for Froude numbers in the 0.5–1.8 range. Well-controlled experiments show large fluctuations of the bedload transport rate under constant supply of water and sediment. Three theoretical or numerical problems have been studied to understand how the input parameters of the model affect the model outputs: (i) the derivation of the stochastic evolution equation for the number of moving particles over fixed plane beds, which leads to exact analytical solutions of the particle activity fluctuation; (ii) the nonlinear simulation of the ensemble-averaged SVEE, which successfully capture the anti-dune regime observed experimentally; (iii) the nonlinear simulation of the stochastic SVEE, which is considered a major challenge.

From a qualitative viewpoint, the solution to the stochastic SVEE captures all of the physical phenomena observed experimentally. The chaotic fluctuations of the sediment transport rate reflect the complex interactions between stochastic fluctuations of particle displacements (on the particle scale) and the nonuniform water flow resulting from bed form development. The part played by particle diffusion is also evaluated by running numerical simulations. Contrary to common belief (within the computation hydraulics community), including diffusion in the evolution equation for  $N$  improves the predictive capability of the model, and must be considered relevant process.

We also note that deterministic approaches (referred to as non-equilibrium sediment transport equations) develop monochromatic oscillations that do not reflect the chaotic pattern observed in real time series of sediment transport rate. In mountain streams, stochastic models perform thus better than these approaches at capturing not only the average sediment transport rate, but also its standard deviation and higher-order moments.