

A generalized Nadai failure criterion for both crystalline and clastic rocks based on true triaxial tests

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The UW true triaxial testing system enables the application of independent compressive loads to cuboidal specimens (19×19×38 mm) along three principal directions. We used the apparatus to conduct extensive series of experiments in three crystalline rocks (Westerly granite, KTB amphibolite, and SAFOD granodiorite) and three clastic rocks of different porosities [TCDP siltstone (7%), Coconino sandstone (17%), and Bentheim sandstone (24%)]. For each rock, several magnitudes of σ_3 were employed, between 0 MPa and 100-160 MPa, and for every σ_3 , σ_2 was varied from test to test between $\sigma_2 = \sigma_3$ and $\sigma_2=(0.4 \text{ to } 1.0) \sigma_1$. Testing consisted of keeping σ_2 and σ_3 constant, and raising σ_1 to failure ($\sigma_{1,peak}$).

The results, plotted as $\sigma_{1,peak}$ vs. σ_2 for each σ_3 used, highlight the undeniable effect of σ_2 on the compressive failure of rocks. For each level of σ_3 , the lowest σ_2 tested ($\sigma_2 = \sigma_3$) yielded the data point used for conventional-triaxial failure criterion. However, for the same σ_3 and depending on σ_2 magnitude, the maximum stress bringing about failure ($\sigma_{1,peak}$) may be considerably higher, by as much as 50% in crystalline rocks, or 15% in clastic rocks, over that in a conventional triaxial test. An important consequence is that use of a Mohr-type criterion leads to overly conservative predictions of failure.

The true triaxial test results demonstrate that a criterion in terms of all (three principal stresses is necessary to characterize failure. Thus, we propose a ‘Generalized Nadai Criterion’ (GNC) based on Nadai (1950), i.e. expressed in terms of the two stress invariants at failure (f), $\tau_{oct,f} = \beta \sigma_{oct,f}$, where $\tau_{oct,f} = 1/3[(\sigma_{1,peak} - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_{1,peak})^2]^{0.5}$ and $\sigma_{oct,f} = (\sigma_{1,peak} + \sigma_2 + \sigma_3)/3$, and β is a function that varies from rock to rock. Moreover, the criterion depends also on the relative magnitude of σ_2 , represented by a parameter $b = (\sigma_2 - \sigma_3)/(\sigma_{1,peak} - \sigma_3)$. For each octahedral shear stress at failure ($\tau_{oct,f}$), the lowest mean stress ($\sigma_{oct,f}$) causing failure is when $b = 0$ (or when $\sigma_2 = \sigma_3$), and the magnitude of $\sigma_{oct,f}$ increases with the b value, reaching the highest mean stress at failure when $b = 1$ (or when $\sigma_2 = \sigma_{1,peak}$). Since the number of b parameters tested for each $\tau_{oct,f}$ magnitude is finite, interpolation between adjacent tested b values should yield fairly accurate failure criteria even in cases where the respective b value has not been tested.

In the crystalline rocks the criterion $\tau_{oct,f} = \beta (\sigma_{oct,f})$ is represented by a power-law function. So for example, in Westerly granite, the GNC for the two extreme b values tested are:

$$\tau_{oct,f} = 2.71(\sigma_{oct,f})^{0.85} \text{ (for } b=0) \text{ and } \tau_{oct,f} = 1.57(\sigma_{oct,f})^{0.89} \text{ (for } b=0.4)$$

In the clastic rocks $\tau_{oct,f} = \beta (\sigma_{oct,f})$ is best fitted by a second-order polynomial equation. So for example, in Bentheim sandstone, the GNC for $b=0$ and $b=1$, respectively, are:

$$\tau_{oct,f} = 2.98 + 1.139\sigma_{oct,f} - 0.0036\sigma_{oct,f}^2 \text{ and } \tau_{oct,f} = -5.82 + 0.836\sigma_{oct,f} - 0.0017\sigma_{oct,f}^2$$