

We investigate the capability of the point-mass modeling technique to assess ice-mass variations in small-scale alpine regions from space-borne gravimetric data. To this end, we simulate point-masses in the region of interest, which we try to recover by means of the proposed methodology. In order to solve for the unknown parameters, we make use of genetic algorithms. The insight of this study will be of importance for future work when it comes to the analysis of real satellite data.

## POINT-MASS MODELING

The point-mass modeling (PMM) technique relates gravity disturbances in space to mass changes on the Earth's surface by

$$\delta g(r_i, \lambda_i, \varphi_i) = G \sum_{j=1}^p \frac{r_i - a \cos \psi_{i,j}}{(a^2 + r_i^2 - 2ar_i \cos \psi_{i,j})^{3/2}} \times \delta m_j, i = 1, \dots, q,$$

with the spherical distance  $\psi_{i,j}$ .

## PARAMETER ESTIMATION

The functional model reads  $\mathbf{Ax} = \mathbf{y}$  and is solved for  $\mathbf{x}$  using least-squares adjustment (LSA) or genetic algorithms (GA). The problem is ill-posed and requires regularization (parameter  $\alpha$ ).

- LSA:  $\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A} + \alpha \mathbf{I})^{-1} \mathbf{A}^T \mathbf{y}$
- GA:  $F(\mathbf{x}) = (\mathbf{Ax} - \mathbf{y})^T (\mathbf{Ax} - \mathbf{y}) + \alpha \mathbf{x}^T \mathbf{x} = \min_{\mathbf{x}}$

## ANALYSIS APPROACHES

- Fixed PMM:  $\mathbf{x} = [\delta m_1, \dots, \delta m_p]$
- Free PMM:  $\mathbf{x} = [\delta m_1, \dots, \delta m_p, \lambda_1, \dots, \lambda_p, \varphi_1, \dots, \varphi_p]$

Only mass changes  $\delta m_j$  are considered within the fixed PMM approach (yielding a linear functional model). Estimating the point-mass locations  $(\lambda_j, \varphi_j)$  together with the magnitudes  $\delta m_j$  results in a highly non-linear optimization problem denoted as the free PMM approach.

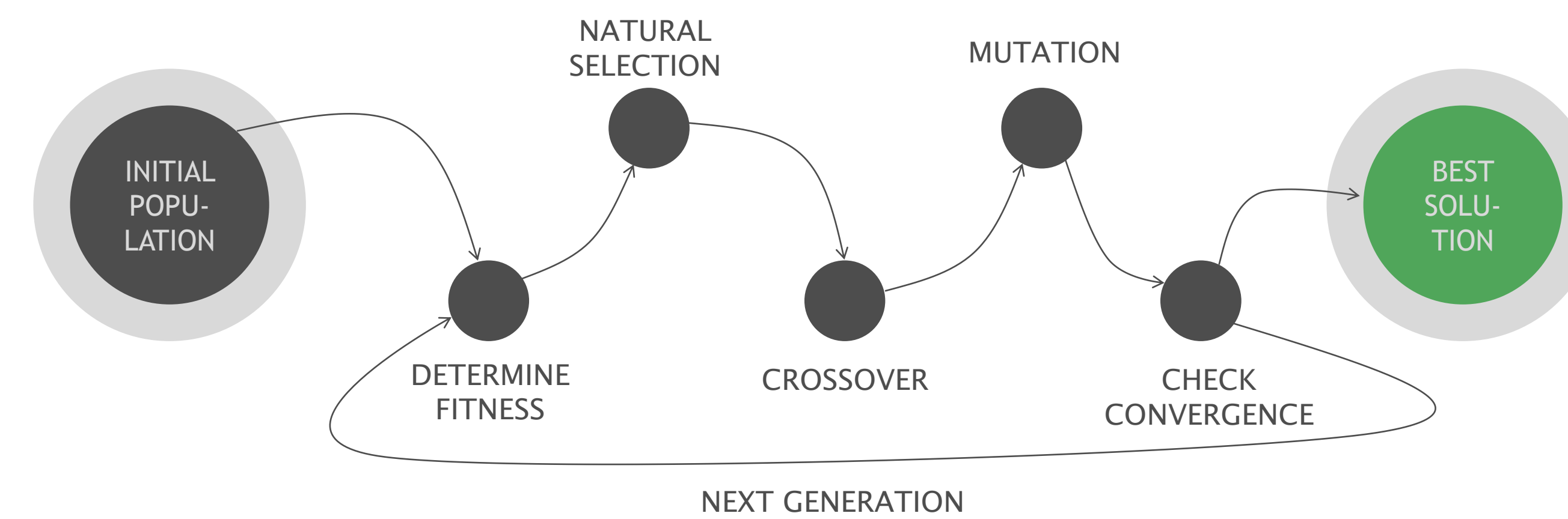
# ON THE USE OF THE POINT-MASS MODELING TECHNIQUE FOR ASSESSING ICE-MASS VARIATIONS IN ALPINE GLACIER SYSTEMS

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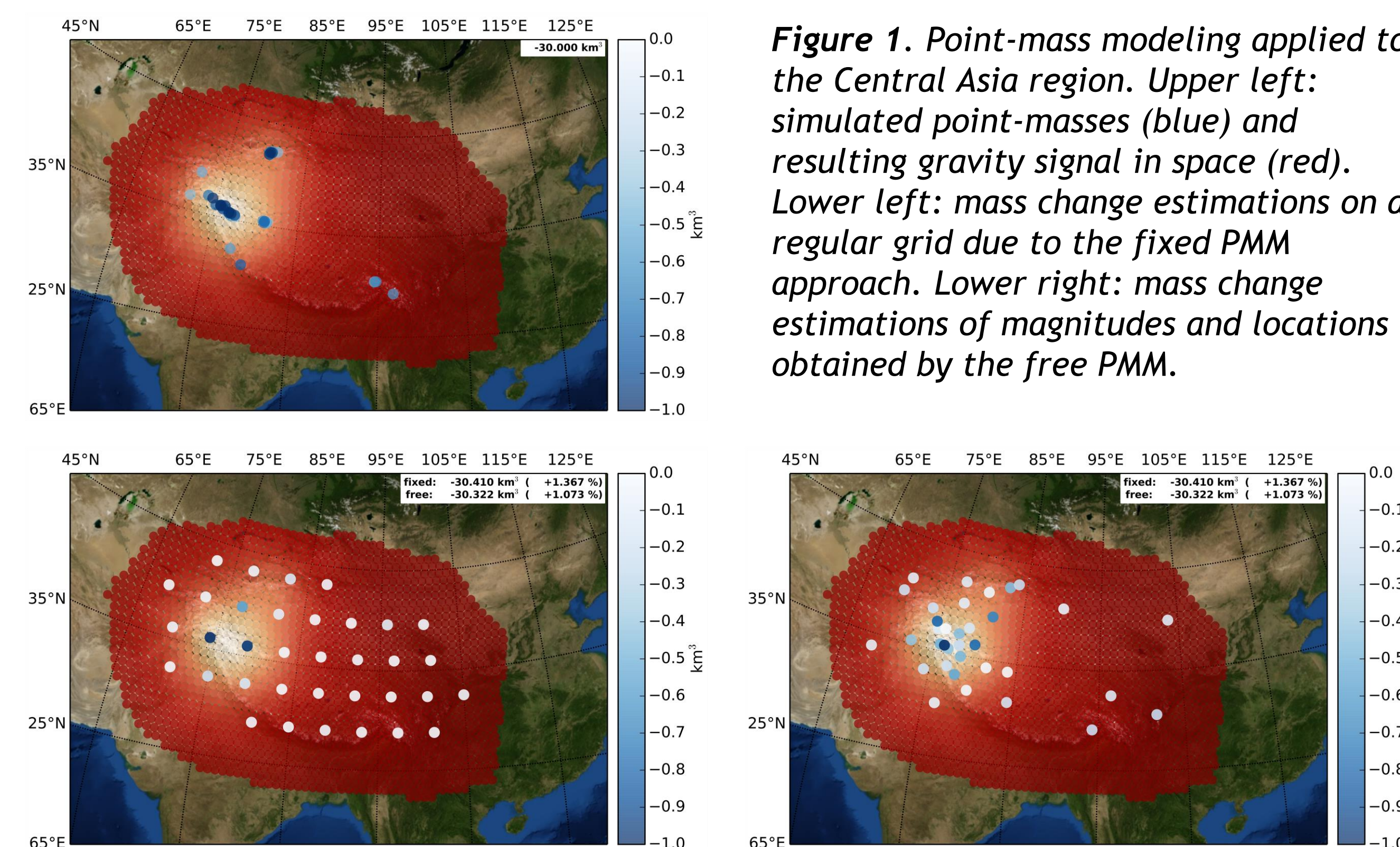
## GENETIC ALGORITHMS

Based on the evolutionary idea of natural selection, this heuristic search algorithm aims to detect the global minimum (or maximum) of an optimization problem without any a priori knowledge. The flowchart diagram below illustrates the process of evolving a set of random solutions (the population) towards better solutions.



## EXAMPLE: CENTRAL ASIA

We demonstrate the viability of the proposed methodology by the example of the Central Asia region. The simulation setup and the numerical results are stated in Table 1. Figure 1 illustrates the input point-masses and the results of the fixed PMM and the free PMM at the end of the GA search.



**Figure 1.** Point-mass modeling applied to the Central Asia region. Upper left: simulated point-masses (blue) and resulting gravity signal in space (red). Lower left: mass change estimations on a regular grid due to the fixed PMM approach. Lower right: mass change estimations of magnitudes and locations obtained by the free PMM.

## SIMULATION TECHNIQUE

1. Selection of the region of interest (ROI).
  - Delineation in a GUI.
2. Simulation of point-masses based on WGI data.
  - Spatial information of glaciers.
3. Computation of the resulting gravity signal at satellite altitude.
  - On a grid covering the ROI and its nearby regions.
4. Estimation of unknown parameters (global optimization).
  - Point-masses (magnitudes and coordinates)
  - Additionally: regularization parameter  $\alpha$
5. Comparison of input point-masses with results.
  - Total mass variation
  - Individual point-masses

**Table 1.** Numerical results of the investigation of the Central Asia region. We simulated 35 point-masses and 1641 observations at an altitude of 500 km. The total variation was assumed to be -30 km<sup>3</sup> with a maximum mass change of -3.284 km<sup>3</sup> located at (76.908°, 35.558°).

	Fixed PMM	Free PMM
Total variation	-30.410 km <sup>3</sup> (-1.367 %)	-30.322 km <sup>3</sup> (-1.073 %)
Largest variation	-8.426 km <sup>3</sup> (-156.560 %) (74.250°, 36.000°), (297 km)	-3.320 km <sup>3</sup> (-1.098 %) (76.114°, 35.621°), (88 km)
Regularization	$5.6 \times 10^{-67}$	$2.8 \times 10^{-52}$

## SUMMARY AND OUTLOOK

Point-mass modeling is a promising technique for assessing mass changes from time-variable gravity data. Our results indicate that better approximations for both magnitudes and geometries of simulated mass changes can be obtained by using the free-positioned PMM approach. GA turn out to be a suitable tool for solving this highly non-linear optimization problem. However, runtime is a serious issue. In order to speed up the computations, effort will be put into refining the algorithms, e.g. by combining a truncated GA with a local optimizer (hybrid GA).

This study was conducted as a preparation for the analysis of real GRACE/GOCE data. Trial and error is an essential part of setting up the GA properly and the experiences from this work will facilitate future investigations tremendously.

## ACKNOWLEDGEMENTS AND REFERENCES

